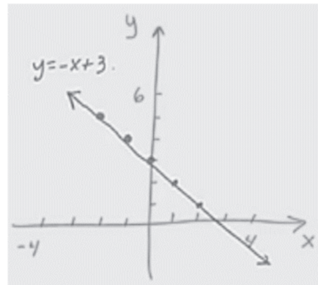
CHAPTER 1 Graphs

Section 1.1: Graphing Utilities; Introduction to Graphing Equations

p. 1. Cartesian; *x*; *y*; origin; ordered pair; quadrants; abscissa; ordinate

Example 1.

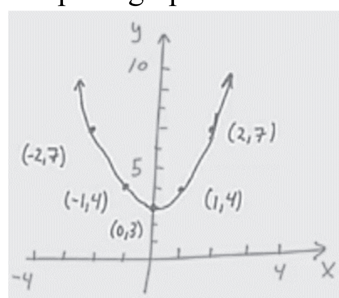
x	*y*	*(x, y)*
-2	$y = -(-2) + 3 = 5$	(-2, 5)
-1	4	(-1, 4)
0	3	(0, 3)
1	2	(1, 2)
2	1	(2, 1)



p. 1. Experience will guide you as to how many points to choose. Plot enough points in order to show the interesting features, which will produce a complete graph.

Example 2.

x	*y*	*(x, y)*
-2	7	(-2, 7)
-1	4	(-1, 4)
0	3	(0, 3)
1	4	(1, 4)
2	7	(2, 7)



p. 2. smallest; largest; tick; smallest; largest; tick

Example 3. *y*; $y = -2x^2 + 12$; $y =$; negative; subtraction; standard; -10; 10; 1; -10; 10; 1; zoom; ZStandard; interesting.

Example 4. Answers will vary depending on what type of graphing utility is used.

p. 4. (3, 0); (0, 3); intercept; *x*-intercept; *y*-intercept.

Example 5. **(a)** $(-3,0);(0,3);\left(\frac{3}{2},0\right);\left(0,-\frac{4}{3}\right);(4.5,0);(0,-3.5)$ **(b)** $(-3,0);\left(\frac{3}{2},0\right);(4.5,0)$

(c) $\left(0,-\frac{4}{3}\right);(0,-3.5)$

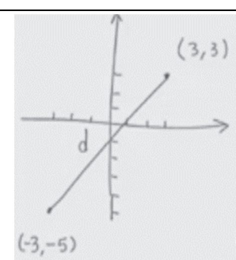
Example 6. **(a)** (-2, 0); (0, 1) **(b)** -2 **(c)** 1

Example 7. Answer will vary.

Section 1.2: The Distance and Midpoint Formulas

Exploration 1. (3, -5); 6, |-3 – 3| = 6; 8, |-3 - 5| = 8; Pythagorean, 10;

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



p. 7. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

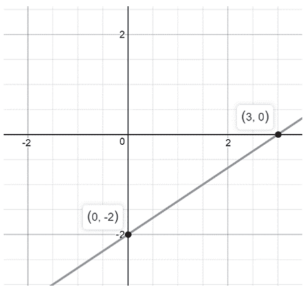
Example 1. $\sqrt{17}$

p. 8. halfway; $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Example 2. (2, 1)

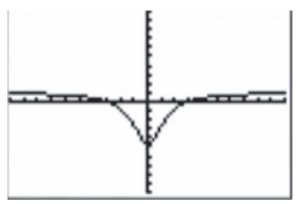
Section 1.3: Intercepts; Symmetry; Graphing Key Equations

p. 9. 0; 0; 0, x; 0, y
Example 1. Intercepts at (0, -2) and (3, 0)

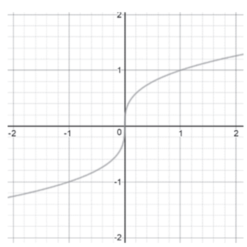


p. 9. $(x,-y);(-x,y);(-x,-y)$

Example 2. (a) $y = 0$, x - intercept is ± 3 **(b)** $x = 0$, y - intercept is $-\dfrac{9}{2}$ **(c)** y by -y, $-y = \dfrac{x^2-9}{x^2+2}$, no x – axis symmetry **(d)** x by -x, $y = \dfrac{x^2-9}{x^2+2}$, y – axis symmetry **(e)** x by -x and y by -y, $-y = \dfrac{x^2-9}{x^2+2}$, no origin symmetry **(f)**



Example 3. Answers will vary



Section 1.4: Solving Equations Using a Graphing Utility

p. 11. Equal; sides; solutions; roots; 0; $\{\text{expression in } x\}$; x-intercept

Example 1. $\{-2.13, -0.20, 2.33\}$

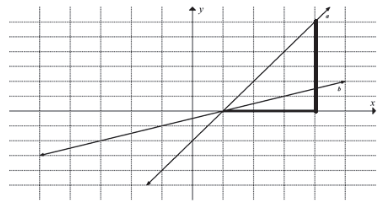
p. 11.
$\{\text{expression in } x \text{ on the left side of the equation}\};\{\text{expression in } x \text{ on the left side of the equation}\}$
$x-$coordinate

Example 2. $\{2\}$

Section 1.5: Lines

p. 12. $m=\dfrac{y_2-y_1}{x_2-x_1}$, $m=\dfrac{y_2-y_1}{x_2-x_1}=\dfrac{\text{rise}}{\text{run}}=\dfrac{\text{Change in } y}{\text{Change in } x}=\dfrac{\Delta y}{\Delta x}$

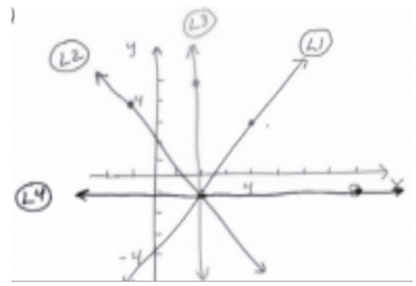
Exploration 1. Larger;



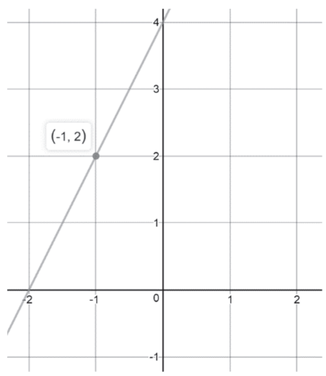
Example 1. (a) 2; the order does not matter as long as the difference in x's is in the denominator and the difference in y's is in the numerator

Example 2. (a) 2 **(b)** $-\dfrac{5}{3}$ **(c)** undefined **(d)** 0

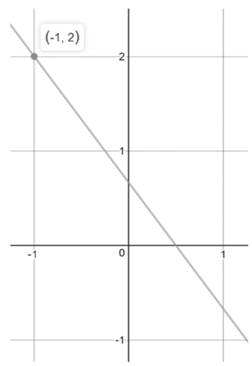
Upward from left to right (L1); downward from left to right (L2); horizontal (L3); vertical (L4)



Example 3. (a)



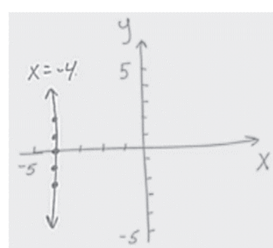
(b)



Example 4.

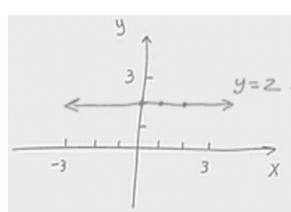
(a)

x	*y*	*(x, y)*
-4	-2	(-4, -2)
-4	-1	(-4, -1)
-4	0	(-4, 0)
-4	1	(-4, 1)
-4	2	(-4, 2)



(b)

x	*y*	*(x, y)*
-2	2	(-2, 2)
-1	2	(-1, 2)
0	2	(0, 2)
1	2	(1, 2)
2	2	(2, 2)



p. 14. $x=a$, a is the x – intercept; $y=b$, b is the y - intercept

Exploration 2. $m=\dfrac{y-y_1}{x-x_1}; y-y_1=m(x-x_1)$; point, slope

p. 15. $y-y_1=m(x-x_1)$

Example 5. $y+3=5(x-4)$

Example 6. $y=4$

Example 7. (a) $m=\frac{1}{3}$ **(b)** $y-1=\frac{1}{3}(x+2)$ or $y-4=\frac{1}{3}(x-7)$

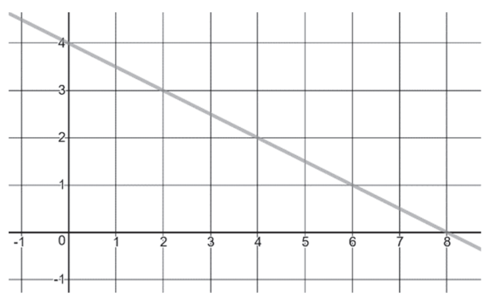
Exploration 3. $y-b=0(x-0)$ or $y=b$; y; $y=mx+b$; a line written in this form identifies both the slope and the y – intercept

p. 16. $y=mx+b$

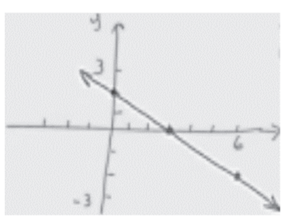
Example 8. $y=\frac{1}{3}x+\frac{5}{3}$

Example 9. (a) $y=-\frac{1}{2}x+4$ **(b)** slope = $-\frac{1}{2}$, y – intercept = 4

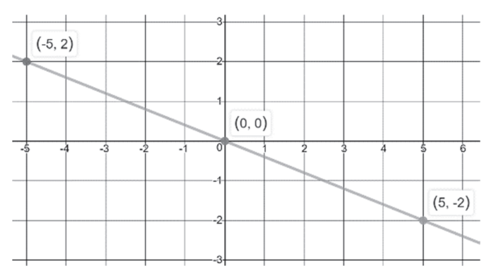
(c)



Example 10.



Example 11. Intercepts are both (0, 0), therefore a couple more points are needed in order to graph. Let x = -5 to get (-5, 2) or x = 5 to get (5, -2).



p. 17. $Ax+By=C$; equal; y – intercepts; x – intercepts

Example 12. same; different; $L_1: y=-3x+\frac{9}{2}; L_2: y=-3x-3$; lines are parallel

Example 13. (a) slope = -4 **(b)** y = -4x + 5

p. 18. right angle or 90°; -1

Example 14. -1; negative reciprocals; $L_1 : y = -\frac{1}{3}x - 5; L_2 : y = 3x - 1$; lines are perpendicular

Example 15. (a) slope = $\frac{2}{5}$ **(b)** slope = $-\frac{5}{2}$, $y + 3 = -\frac{5}{2}(x - 2)$ **(c)** $y = -\frac{5}{2}x + 2$

Section 1.6: Circles

p. 19. radius; center

Exploration 1. Label center (h, k) and radius r.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$r = \sqrt{(x - h)^2 + (y - k)^2}$

$r^2 = (x - h)^2 + (y - k)^2$

p. 19. radius; center; $r^2 = (x - h)^2 + (y - k)^2$

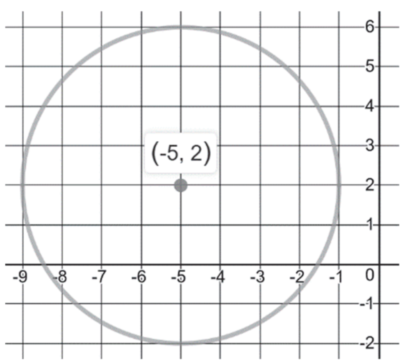
Example 1. (a) $r^2 = x^2 + y^2$ **(b)** $1 = x^2 + y^2$

p. 20. $x^2 + y^2 = r^2$; $x^2 + y^2 = 1$

Example 2. $(x + 5)^2 + (y - 2)^2 = 16$; answers will vary

Example 3. (a) (-5, 2) **(b)** 4

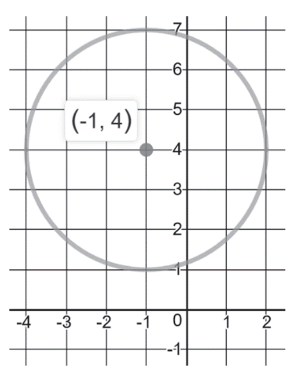
(c)



Exploration 2. $x^2 + 10x + 25 + y^2 - 4y + 4 = 16; x^2 + y^2 + 10x - 4y + 13 = 0$

p. 21. $x^2 + y^2 + ax + by + c = 0$

Example 4. (a) $(x + 1)^2 + (y - 4)^2 = 9$ **(b)** (-1, 4) **(c)** 3 **(d)**



CHAPTER 2 Functions and Their Graphs

Section 2.1: Functions

p. 22. Correspondence; domain; range; corresponds; depends on; $x \to y$; equations, graphs, mapping, ordered pairs; exactly one element; the set of all input values; the set of all output values; Yes elements in the range can be repeated in a function. The definition of a function only states that each element in the *domain* can only be mapped to exactly one element in the range. The range values can be repeated.

Example 1. (a) Function; domain: {No High School Diploma, High School Diploma, Some College, College Graduate}; range: {3.4%, 5.4%, 5.9%, 7.7%} **(b)** Not a function **(c)** Function; domain: {-2, 2, 3, 4}; range: {-1, -2, 1, 3} **(d)** Not a function

Example 2. (a) Function **(b)** Not a function

p. 23. the domain of the function; exactly one output (which may be repeated for different inputs); independent variable; dependent variable.

Example 3. (a) name: *g*; independent variable: *x*; dependent variable: *y* **(b)** name: *f*; independent variable: *x*; dependent variable: *y* **(a)** name: *r*; independent variable: *n*; dependent variable: *p*

Example 4. (a) -21 **(b)** $-3x^2+2x-21$ **(c)** $-3x^2-2x$ **(d)** $3x^2-2x$ **(e)** $-3x^2-16x-21$

p. 24. implicitly; explicitly

Example 5. Implicit functions: $3x+y=5$ and $xy=4$

p. 25. $\dfrac{f(x+h)-f(x)}{h}, h \neq 0$

Example 6. (a) -2 **(b)** $4x+2h-5$

p. 25. real numbers; the denominator is zero; these numbers must be excluded because dividing by zero is undefined; negative; these numbers must be excluded because they are undefined.

Example 7. (a) $\{x \mid x \neq -1, x \neq 3\}$ **(b)** $(-\infty, \infty)$ **(c)** $\left(-\infty, \dfrac{3}{2}\right]$

Example 8. $\{w \mid 0 < w < 50\}$

p. 26. $f(x)-g(x)$; numbers that are in the domains of both *f* and *g*; $f(x)+g(x)$; numbers that are in the domains of both *f* and *g*; $f(x)\cdot g(x)$; numbers that are in the domains of both *f* and *g*; $\dfrac{f(x)}{g(x)}$; numbers *x* for which $g(x) \neq 0$ and that are in the domains of both *f* and *g*

Example 9. (a) $-x+2; (-\infty,\infty)$ **(b)** $5x-10; (-\infty,\infty)$ **(c)** $-6x^2+24x-24; (-\infty,\infty)$ **(d)** $-\dfrac{2}{3}; \{x \mid x \neq 2\}$ **(e)** 1 **(f)** -5 **(g)** -54 **(h)** $-\dfrac{2}{3}$

Example 10. (a) $\dfrac{x^2-x+4}{(x-2)(x+4)}: \{x \mid x \neq -4, x \neq 2\}$ **(b)** $\dfrac{-x^2+3x+4}{(x-2)(x+4)}: \{x \mid x \neq -4, x \neq 2\}$ **(c)** $\dfrac{x}{(x-2)(x+4)}: \{x \mid x \neq -4, x \neq 2\}$ **(d)** $\dfrac{x+4}{x(x-2)}: \{x \mid x \neq -4, x \neq 0, x \neq 2\}$

Section 2.2: The Graph of a Function

p. 28. at most one point

Example 1. (a) Function; $D:(-\infty,\infty); R:[0,\infty)$ **(b)** Function; $D:(-\infty,\infty); R:(-\infty,\infty)$ **(c)** Not a function; $D:[0,\infty); R:(-\infty,\infty)$ **(d)** Not a function; $D:[-3,3]; R:[-3,3]$

p. 28. The vertical line test indicates that every input is assigned only one output.

p. 29. $y=f(x); -2; 7; (5,8)$

Example 2. (a) $f(0)=4; f\left(\frac{3\pi}{2}\right)=0; f(3\pi)=-4$ **(b)** $\{x\,|\,0\le x\le 4\pi\}$ **(c)** $\{y\,|\,-4\le y\le 4\}$

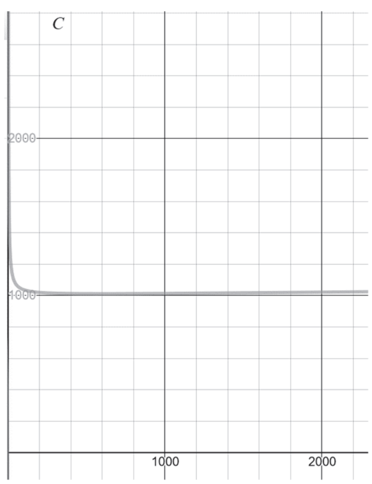
(d) $(0,4),\left(\frac{\pi}{2},0\right),\left(\frac{3\pi}{2},0\right),\left(\frac{5\pi}{2},0\right),\left(\frac{7\pi}{2},0\right)$ **(e)** 4 **(f)** $\{\pi,3\pi\}$

(g) $0\le x<\frac{\pi}{2}, \frac{3\pi}{2}<x<\frac{5\pi}{2}, \frac{7\pi}{2}<x\le 4\pi$

Example 3. (a) Yes **(b)** $\left(2,\frac{2}{3}\right)$ **(c)** $(-2,2)$ **(d)** $\{x\,|\,x\ne -1\}$ **(e)** $(0,0)$ **(f)** $(0,0)$

Example 4. (a) \$1020; ≈\$1014; **(b)** $\{x\,|\,x>0\}$

(c) **(e)** 600 mph



Section 2.3: Properties of Functions

p. 32. $f(x); (-x,y);$ symmetric; $y; -f(x); (-x,-y);$ origin

Example 1. (a) Even **(b)** Neither **(c)** Odd

Example 2. (a) Even **(b)** Neither **(c)** Odd **(d)** Even

p. 33. $f(x_1)<f(x_2); f(x_1)>f(x_2);$ equal; As x gets bigger, the values of the function get smaller; As x gets bigger or smaller, the values of the function remain constant.

Example 3. (a) $(-4,0)$ **(b)** $(-6,-4)\cup(3,6)$ **(c)** $(0,3)$

p. 35. $f(c)\ge f(x)$; $f(c)\le f(x)$

Example 4. (a) 1 **(b)** 2 **(c)** -1 and 3 **(d)** 1 and 0

Example 5. Local maxima: 2.4; Local Minima: -4.4; Increasing: $(-2,-0.707)\cup(0.707,2)$**;**

Decreasing: $(-0.707,0.707)$

p. 36. $f(x)\le f(u);u;f(x)\ge f(v);v$

Example 6. Absolute maximum: none; Absolute minimum: 0; Absolute maximum: 2; Absolute minimum: none; Absolute maximum: none; Absolute minimum: -4; Absolute maximum: none; Absolute minimum: 0

p. 37. Absolute maximum; absolute minimum

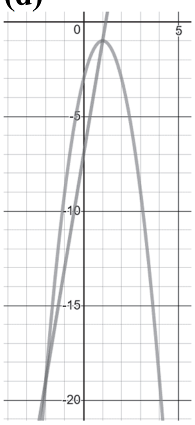
Example 7. No because the fourth graph is not continuous on that interval.

p. 37. $\frac{\triangle y}{\triangle x};\frac{f(b)-f(x)}{b-a};a\ne b;$ slope

Example 8. (a) $\frac{1}{2}$ **(b)** $\frac{3}{2}$ **(c)** $\frac{5}{2}$

Example 9. (a) $-2(x-4)$ **(b)** -6 **(c)** $y=6x-7$

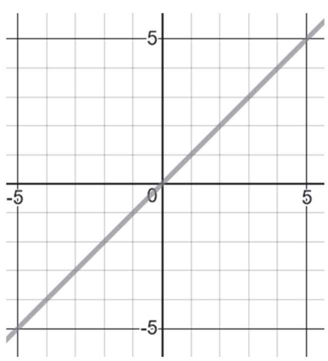
(d)



Section 2.4: Library of Functions; Piecewise-defined Functions

Exploration 1:
Identity Function:

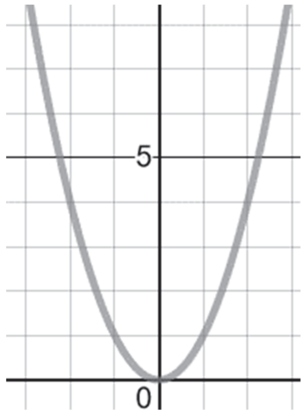
x	*y*
–2	–2
–1	–1
0	0
1	1
2	2



Properties	
Domain:	$(-\infty,\infty)$
Range:	$(-\infty,\infty)$
x-intercept(s):	$(0,0)$
y-intercept:	$(0,0)$
Symmetry:	Origin
Interval the function is decreasing:	None
Interval the function is increasing:	$(-\infty,\infty)$
Local maxima/Minima:	None
Absolute maxima/Minima:	None

Square Function:

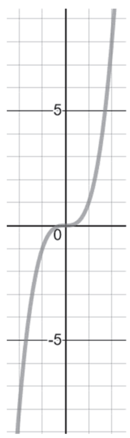
x	y
–2	4
–1	1
0	0
1	1
2	4



Properties	
Domain:	$(-\infty, \infty)$
Range:	$[0,\infty)$
x-intercept(s):	$(0,0)$
y-intercept:	$(0,0)$
Symmetry:	y – axis
Interval the function is decreasing:	$(-\infty, 0)$
Interval the function is increasing:	$(0,\infty)$
Local maxima/Minima:	0
Absolute maxima/Minima:	0

Cube Function:

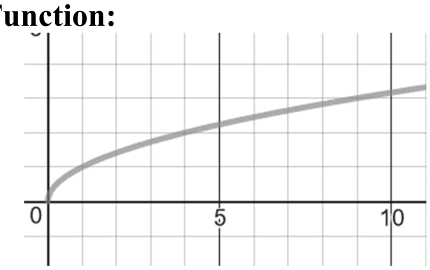
x	y
–2	–8
–1	–1
0	0
1	1
2	8



Properties	
Domain:	$(-\infty, \infty)$
Range:	$(-\infty, \infty)$
x-intercept(s):	$(0,0)$
y-intercept:	$(0,0)$
Symmetry:	Origin
Interval the function is decreasing:	Never
Interval the function is increasing:	$(-\infty, \infty)$
Local maxima/Minima:	None
Absolute maxima/Minima:	None

Square Root Function:

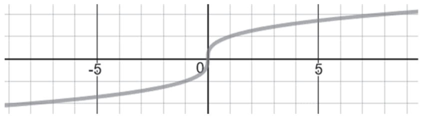
x	y
–4	DNE
–1	DNE
0	0
1	1
4	2



Properties	
Domain:	$[0,\infty)$
Range:	$[0,\infty)$
x-intercept(s):	$(0,0)$
y-intercept:	$(0,0)$
Symmetry:	None
Interval the function is decreasing:	Never
Interval the function is increasing:	$[0,\infty)$
Local maxima/Minima:	None
Absolute maxima/Minima:	0

Cube Root Function:

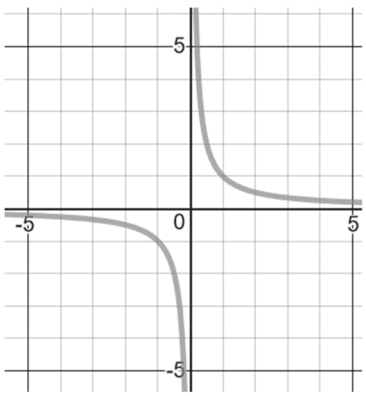
x	*y*
–8	–2
–1	–1
0	0
1	1
8	2



Properties	
Domain:	$(-\infty, \infty)$
Range:	$(-\infty, \infty)$
x-intercept(s):	$(0, 0)$
y-intercept:	$(0, 0)$
Symmetry:	Origin
Interval the function is decreasing:	Never
Interval the function is increasing:	$(-\infty, \infty)$
Local maxima/Minima:	None
Absolute maxima/Minima:	None

Reciprocal Function:

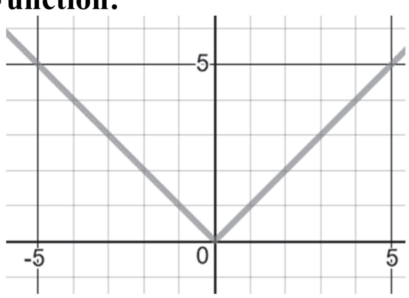
x	*y*
–2	$-\frac{1}{2}$
–1	–1
0	0
1	1
2	$\frac{1}{2}$



Properties	
Domain:	$\{x \mid x \neq 0\}$
Range:	$\{y \mid y \neq 0\}$
x-intercept(s):	None
y-intercept:	None
Symmetry:	Origin
Interval the function is decreasing:	$(-\infty, 0) \cup (0, \infty)$
Interval the function is increasing:	None
Local maxima/Minima:	None
Absolute maxima/Minima:	None

Absolute Value Function:

x	*y*
–2	2
–1	1
0	0
1	1
2	2

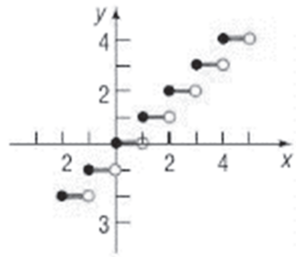


Properties	
Domain:	$(-\infty, \infty)$
Range:	$[0, \infty)$
x-intercept(s):	$(0, 0)$
y-intercept:	$(0, 0)$
Symmetry:	y – axis
Interval the function is decreasing:	$(-\infty, 0)$
Interval the function is increasing:	$(0, \infty)$
Local maxima/Minima:	0
Absolute maxima/Minima:	0

Constant Function: Straight line; *b*

Greatest Integer Function:

x	*y*
–1	–1
$-\frac{1}{2}$	–1
$-\frac{1}{4}$	–1
0	0
$\frac{1}{4}$	0
$\frac{1}{2}$	0
$\frac{3}{4}$	0
1	1
1.5	1
2	2
2.7	2
3	3
3.2	3

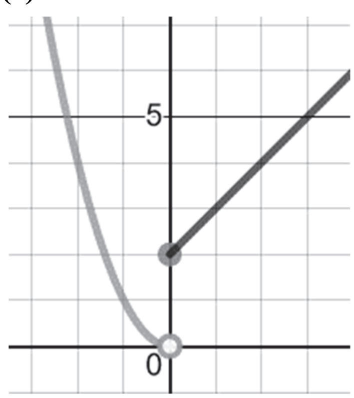


The graph suddenly "steps" from one value to another without taking on any of the intermediate values; The function is discontinuous because it has gaps and holes.

p. 34. equations; domain

Example 1. (a) $f(-2)=4; f(0)=2; f(3)=5$ **(b)** $(-\infty,\infty)$

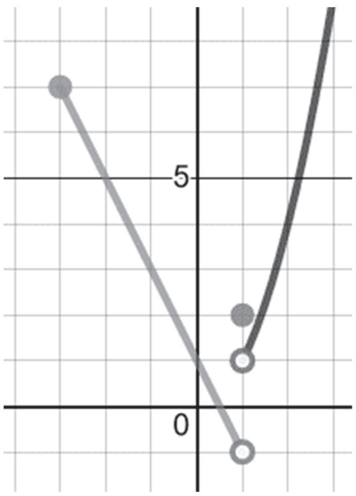
(c)



(d) $(0,\infty)$ **(e)** No

Example 2. (a) $f(-2)=5; f(1)=2; f(2)=4$ **(b)** $[-3,\infty)$ **(c)** $\left(\frac{1}{2},0\right),(0,1)$

(d) $(-1,\infty)$ **(e)** No

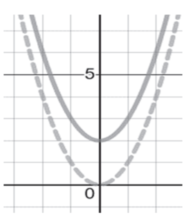


Example 3. (a) \$32.41 **(b)** \$59.30 **(c)** $C(x)=\begin{cases}7.58+0.08275x & \text{if} \quad 0\le x\le 400\\ 15.848+0.06208x & \text{if} \quad x>4\end{cases}$

Section 2.5: Graphing Techniques: Transformations

Exploration 1.

(a)



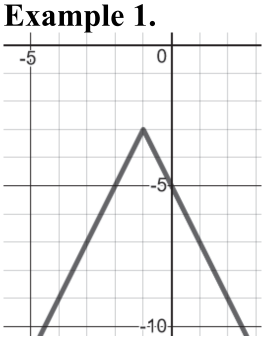
(c) The graph of $y=x^2+2$ is identical to $y=x^2$, except it is shifted vertically up two units.

p. 47. vertically up; vertically down
p. 48. shifted horizontally right; shifted horizontally left
p. 49. stretched; compressed
p. 50. compressed; stretched
Exploration 5. reflection over the x-axis; reflection over the y-axis; reflection over the x-axis; reflection over the y- axis.
p. 51. x; y

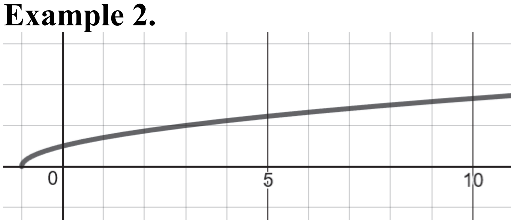
SUMMARY OF GRAPHING TECHNIQUES

To Graph:	Draw the Graph of f and:	Functional Change to $f(x)$
Vertical shifts		
$y = f(x) + k, \quad k > 0$	Raise the graph of f by k units.	Add k to $f(x)$.
$y = f(x) - k, \quad k > 0$	Lower the graph of f by k units.	Subtract k from $f(x)$.
Horizontal shifts		
$y = f(x + h), \quad h > 0$	Shift the graph of f to the left h units.	Replace x by $x + h$.
$y = f(x - h), \quad h > 0$	Shift the graph of f to the right h units.	Replace x by $x - h$.
Compressing or stretching		
$y = af(x), \quad a > 0$	Multiply each y-coordinate of $y = f(x)$ by a. Stretch the graph of f vertically if $a > 1$. Compress the graph of f vertically if $0 < a < 1$.	Multiply $f(x)$ by a.
$y = f(ax), \quad a > 0$	Multiply each x-coordinate of $y = f(x)$ by $\frac{1}{a}$. Stretch the graph of f horizontally if $0 < a < 1$. Compress the graph of f horizontally if $a > 1$.	Replace x by ax.
Reflection about the x-axis		
$y = -f(x)$	Reflect the graph of f about the x-axis.	Multiply $f(x)$ by -1.
Reflection about the y-axis		
$y = f(-x)$	Reflect the graph of f about the y-axis.	Replace x by $-x$.

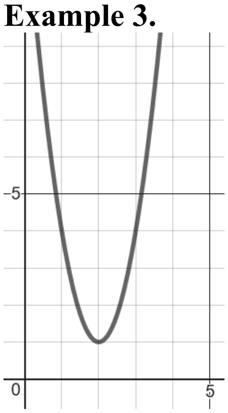
Example 1.



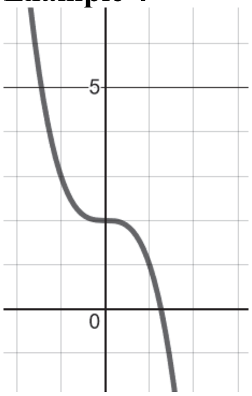
Example 2.



Example 3.



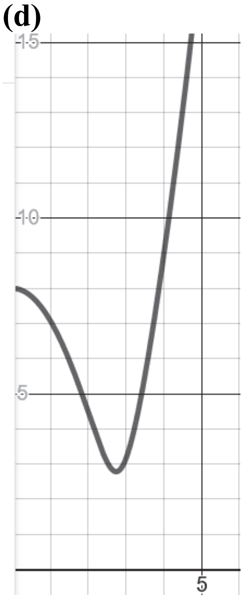
Example 4



Section 2.6: Mathematical Models: Building Functions

Example 1. (a) $d(x)=\sqrt{x^4-15x^2+64}$ **(b)** 8 **(c)** $5\sqrt{2}$

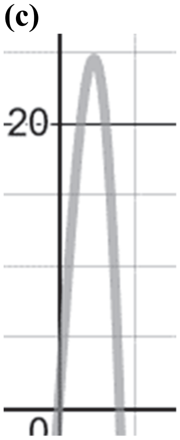
(d)



(e) ≈ 2.74

Example 2. (a) $A(x)=-x^3+16x$ **(b)** $\{x \mid 0 < x < 4\}$

(c)



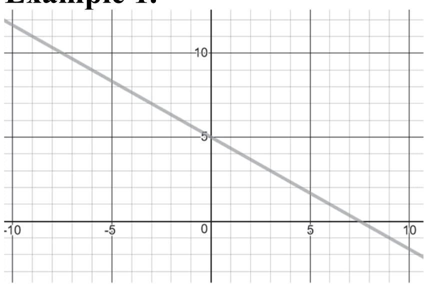
(d) ≈ 2.31**;** This is called an absolute maximum

CHAPTER 3 Linear and Quadratic Functions

Section 3.1: Properties of Linear Functions and Linear Models

p. 57. $f(x) = mx + b$; m; b; the set of all real numbers

Example 1.



p. 57. $f(x) = mx + b; \frac{\Delta y}{\Delta x} = m$

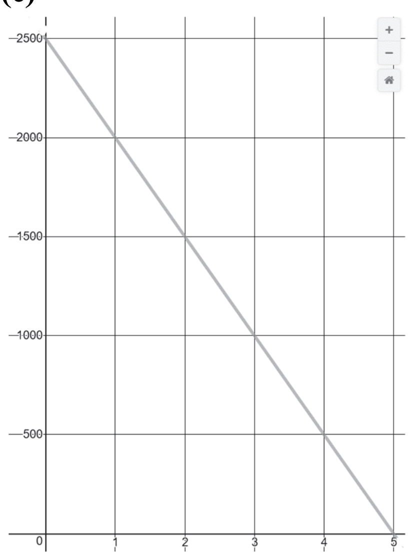
Example 2. (a) Linear; $y = -3x - 2$ **(b)** Not linear

Exploration 1. (a) iv; ii; i; v; vi; iii **(b)** $f(x)=\frac{2}{3}x-2$ and $f(x)=\frac{5}{3}x+2$ are increasing. They both have positive slopes. **(c)** $f(x)=-\frac{2}{3}x-2$ and $f(x)=-\frac{5}{3}x+2$ are decreasing. They both have negative slopes. **(d)** $f(x) = 2$ and $f(x) = -2$ are constant. They both have a slope of zero. **(e)** its slope is positive; its slope is negative; its slope is zero.

p. 59. $f(x) = mx + b$; the value of f at 0.

Example 3. (a) $V(x) = -500x + 2500$ **(b)** $\{x \mid 0 \le x \le 5\}$

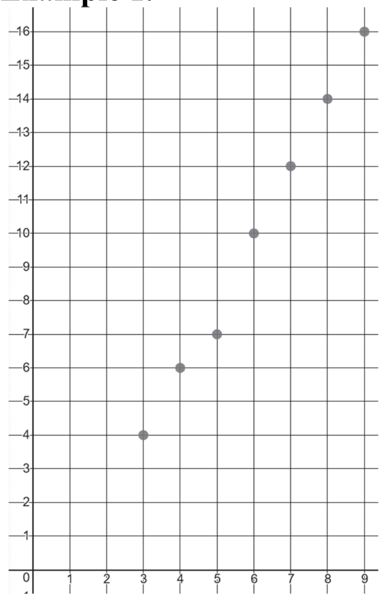
(c)



(d) \$1500 **(e)** -\$500 per year **(f)** 4 years

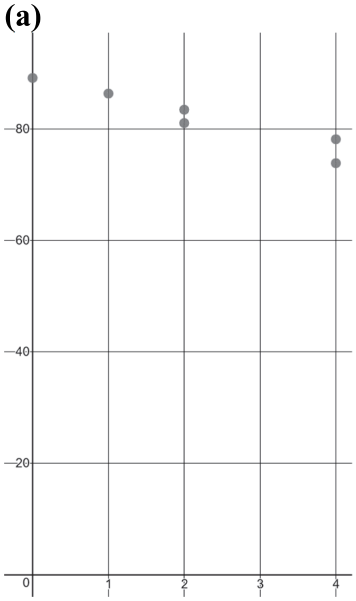
Section 3.2: Building Linear Models from Data

Example 1.



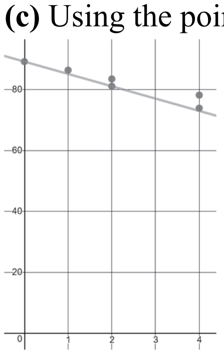
Example 2.

(a)



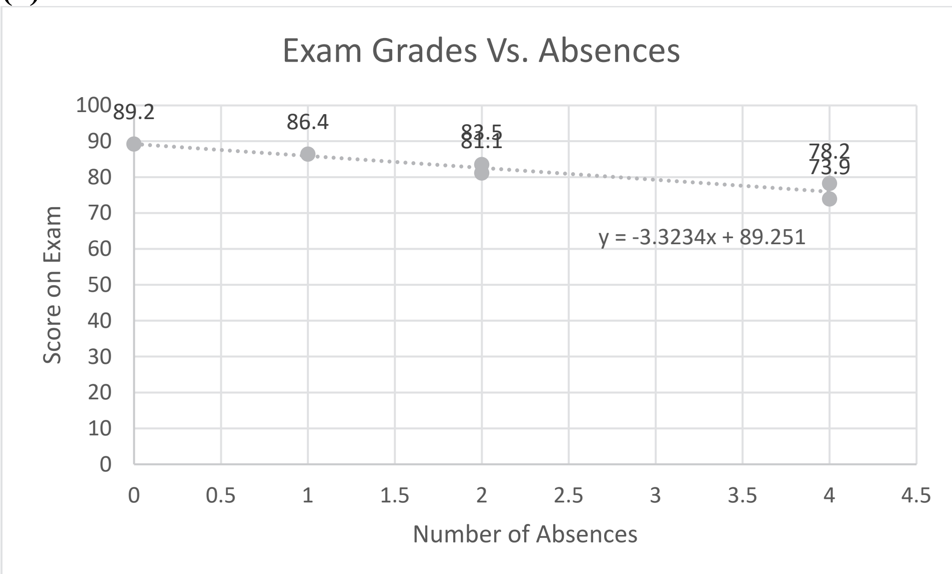
(b) As the number of absences increases, the student scores decrease.

(c) Using the points $(0, 89.2)$ and $(2, 81.1)$; $y = -4.05x + 89.2$

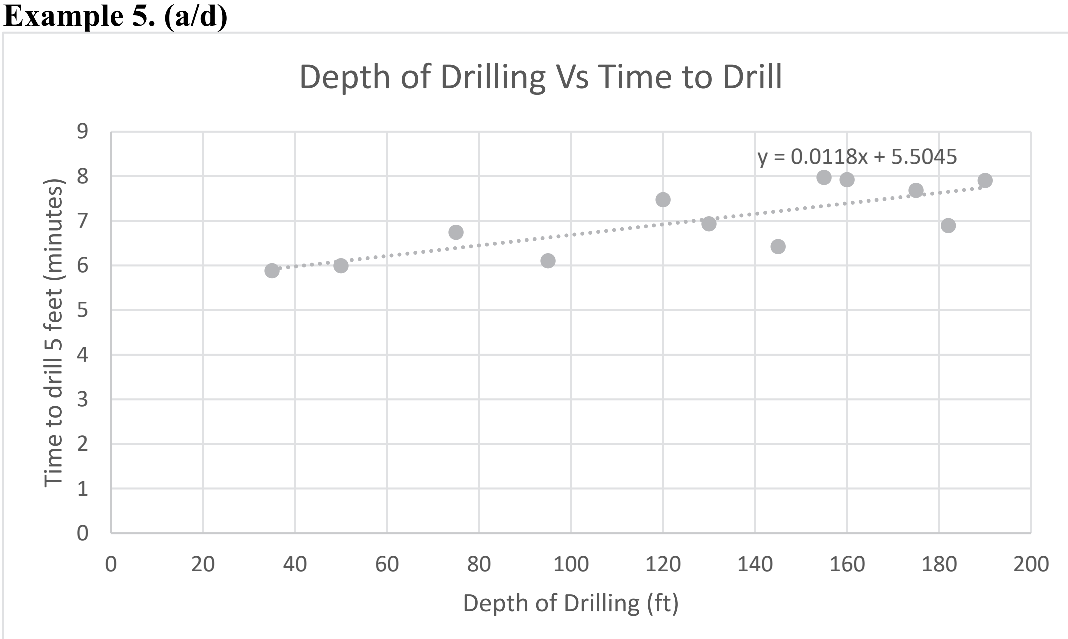


(d) Exam scores decrease by 4.05 points for every 1 absence. **(e)** 77.05

Example 3. Linear; Linear; Nonlinear; Nonlinear; Nonlinear
Example 4. (a) $y = -3.32x + 89.25$
(b)



(c) Exam scores decrease by 3.32 points for every 1 absence
Example 5. (a/d)

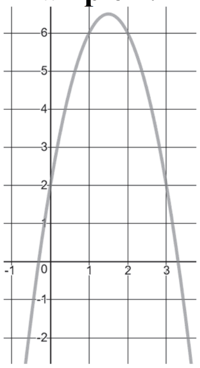


(b) 7.04 minutes **(c)** Above

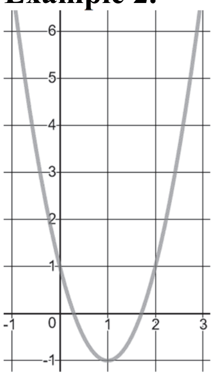
Section 3.3: Quadratic Functions and Their Properties

p. 63. $f(x) = ax^2 + bx + c; a \neq 0;$ the set of all real numbers; If $a \neq 0$ the function would be a linear function; parabola; up; down; vertex; axis of symmetry; This line is called the axis of *symmetry* because it is the line the parabola is symmetric on.

Example 1.



Example 2.



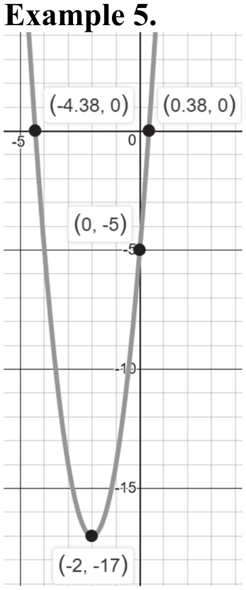
p. 64. $\left(-\dfrac{b}{2a}, f\left(-\dfrac{b}{2a}\right)\right)$; $x=-\dfrac{b}{2a}$; $a>0$; minimum point; $a<0$; maximum point

Example 3. $(-2,-17)$; Opens up

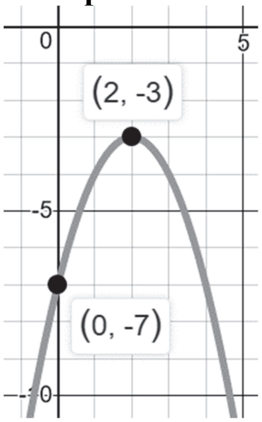
Example 4. $(1,4)$; Opens down

p. 65. two; one; no

Example 5.



Example 6.



p. 66. $f(x)=a(x-h)^2+k$

Example 7. $f(x)=-3(x+2)^2+5$

p. 66. lowest; $f\left(-\frac{b}{2a}\right)$; highest; $f\left(-\frac{b}{2a}\right)$

Example 8. Maximum value; 9

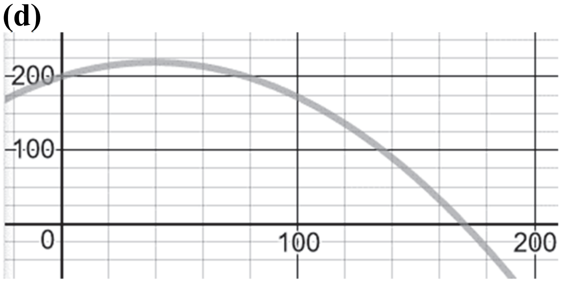
Section 3.4: Build Quadratic Models from Verbal Descriptions and from Data

p. 67. *xp*

Example 1. (a) $R(x)=-\frac{1}{6}x^2+100x$ **(b)** \$13,333.33 **(c)** \$300; \$15,000 **(d)** \$50

Example 2. (a) 39.0625 feet **(b)** 219.53125 feet **(c)** 170 feet

(d)

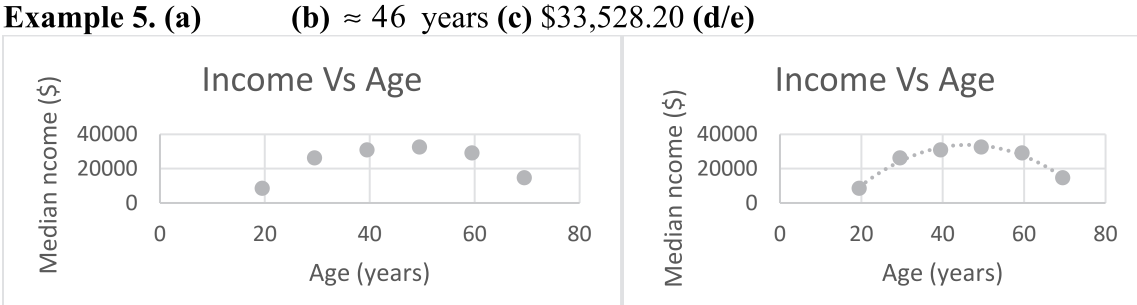


(e) 135.6978 feet

Example 3. (a) $A(w)=-w^2+200w$ **(b)** 100 yards **(c)** 10,000 square yards

Example 4. 18.75 m

Example 5. (a) **(b)** ≈ 46 years **(c)** \$33,528.20 **(d/e)**



Section 3.5: Inequalities Involving Quadratic Functions

Example 1. $\{x \mid -4 < x < 2\}$

Example 2. $\{x \mid x \le -7, x \ge 4\}$

CHAPTER 4 Polynomial and Rational Functions

Section 4.1: Polynomial Functions and Models

p. 73. $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$; coefficients; integer; variable; leading coefficient; degree; descending order according to degree

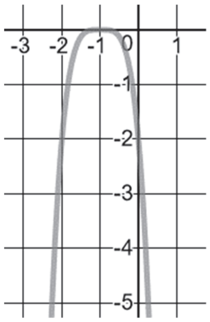
Example 1. (a) Polynomial; 2; $3x^2; -10$ **(b)** Not a polynomial because of a noninteger exponent **(c)** Not a polynomial because of a negative exponent **(d)** Polynomial; 0; -5 **(e)** Polynomial; 3; $6s^3 - 3s; 6s^3; 0$

p. 74. $f(x) = ax^n$

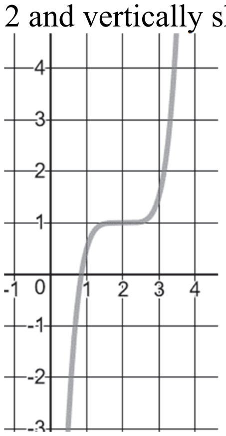
Exploration 1. Even Powered Functions: Yes; Yes $(0,0), (1,1), (-1,1)$; $y-$axis; $(-\infty, \infty)$; $[0, \infty)$; As $x \to \pm\infty$, the graph becomes more steep; As $x \to 0$, the graph becomes more flat.

Odd Powered Functions: Yes; Yes $(0,0), (1,1), (-1,-1)$; origin; $(-\infty, \infty)$; $(-\infty, \infty)$; As $x \to \pm\infty$, the graph becomes more steep; As $x \to 0$, the graph becomes more flat.

Example 2. (a) $f(x)$ takes $y = x^4$ and reflects it over the x- axis, vertically stretches by a factor of 2 and horizontally shift left 1.



(b) $g(x)$ takes $y = x^5$ and vertically compresses it by a factor of $\frac{1}{2}$, horizontally shifts it right 2 and vertically shifts it up 1.



p. 76. cross; touch; above; below; zero-product-; real zero; x-intercept; $x - r$; $f(x) = 0$

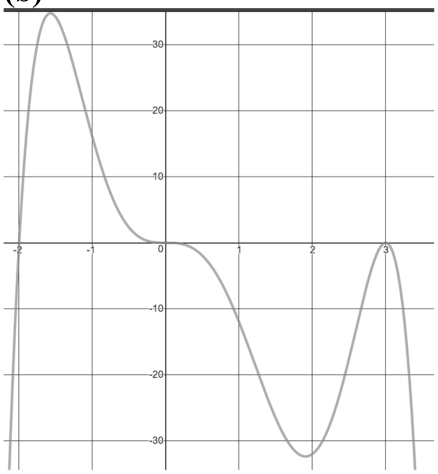
Example 3. (a) $\{-3, 1\}$ **(b)** $\{-1, 2\}$

Example 4. $f(x) = x^3 + 3x^2 - 10x - 24$

p. 77. $(x - r)^m$

Example 5. (a) $(0, 0), (-2, 0), (3, 0)$

(b)

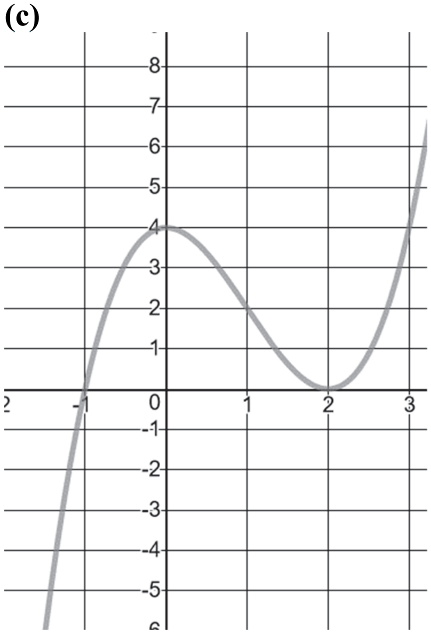


(c) 0 is a zero of multiplicity 3; 3 is a zero of multiplicity 2; -2 is a zero of multiplicity 1

Example 6. (a) $(0, 4), (-1, 0), (2, 0)$

(b) Below the x-axis: $(-\infty, -1)$; above the x-axis: $(-1, 2) \cup (2, \infty)$

(c)



Exploration 2. does not change; touches; changes; crosses

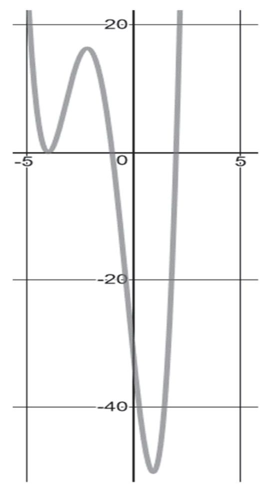
Exploration 3. 1. (a) 2 **(b)** The number of turning points is one less than the degree of the polynomial **2. (a)** 3 **(b)** The number of turning points is one less than the degree of the polynomial

p. 79. 2; 1; $n - 1$; n

Example 7. 2; 1
Example 8. **(a)** The zero of -5 has a multiplicity of 1; The zero of -1 has a multiplicity of 1; The zero of 2 has a multiplicity of 2; The zero of 4 has a multiplicity of 1 **(b)** The end behavior of the graph of the polynomial behaves like the power function
$f(x)=x^5$; as $x\to\infty, f(x)\to\infty$, as $x\to-\infty, f(x)\to-\infty$
Example 9. $f(x)=\frac{1}{2}(x+3)(x+1)^2(x-2)$
Example 10. $f(x)=-2x(x-1)^2$

Section 4.2: Graphing Polynomial Functions; Models

p. 81. end behavior; x- and y-intercepts; zeros; crosses or touches; turning points
Example 1.



1: Behaves like $y=x^4$

2. $(-4,0),(-1,0),(2,0),(0,-32)$

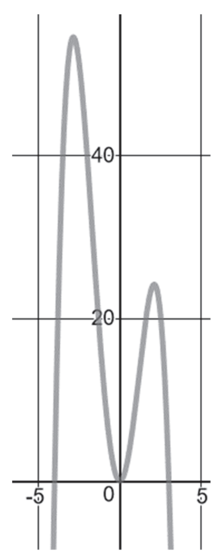
3. Zero of -4, mult. 2, touches; zero of -1, mult. 1, crosses; zero of 2, mult. 1, touches

4. 3

5. Near -4: $f(x)=18(x+4)^2$; Near -1: $f(x)=-27x-27$;

Near 2: $f(x)=108x-324$;

Example 2.



1: Behaves like $y=-x^4$

2. $(0,0),(-4,0),(3,0)$

3. Zero of 0, mult. 2, touches; zero of -4, mult. 1, crosses; zero of 3, mult. 1, crosses

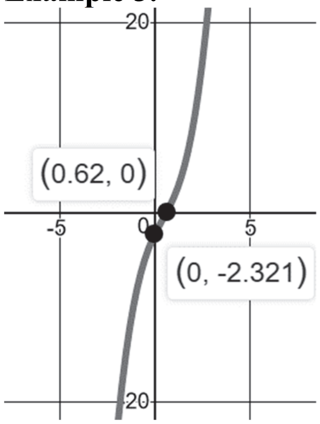
4. 3

5. Near 0: $f(x)=12x^2$; Near -4: $f(x)=112x+448$;

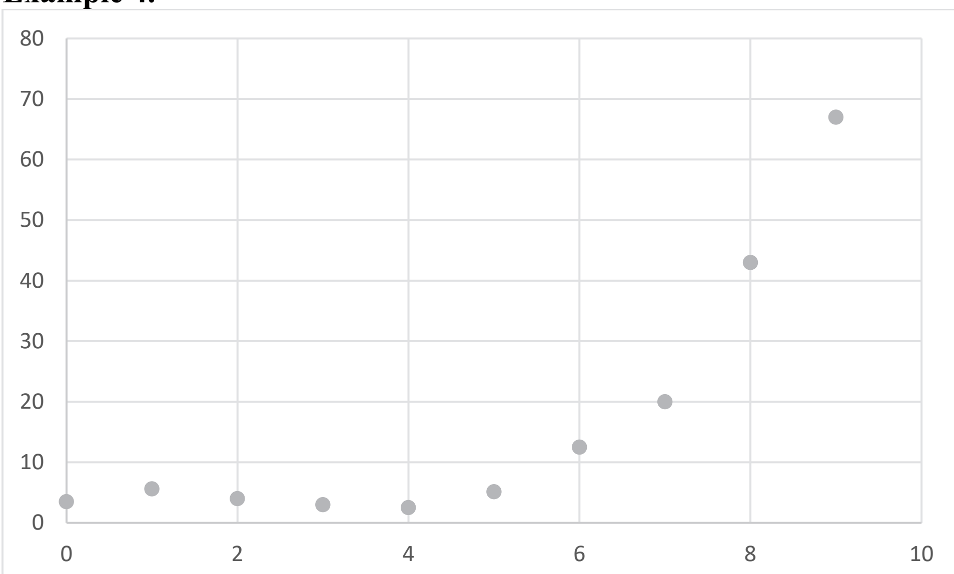
Near 2: $f(x)=-63x+189$;

p. 82. end behavior; x- and y-intercepts of the graph; x-intercept; turning points; range; increasing; decreasing

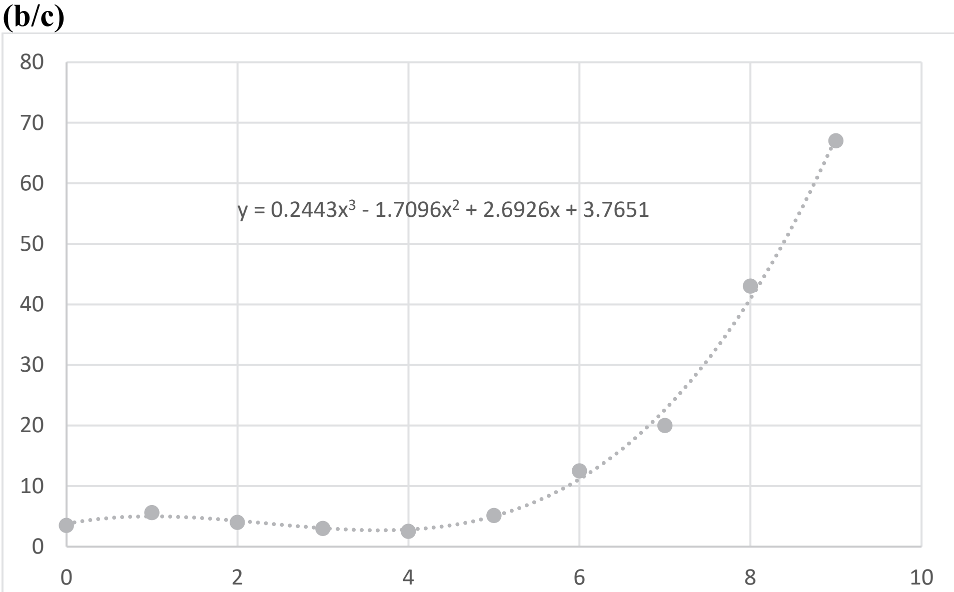
Example 3.



Example 4.



(b/c)



(d) 286.6

Section 4.3: The Real Zeros of a Polynomial Function

Exploration 1. (a) Quotient: 2; divisor: 3; remainder: 2 **(b)** Quotient: 4; divisor: 5; remainder: 2
p. 84. $f(c)$
Example 1: (a) -1 **(b)** 5
p. 84. $f(c)=0; x-c$ is a factor of $f(x); f(c)=0$
Example 2. (a) Yes **(b)** No
p. 85. $f(x); f(-x)$
Example 3. There will be one positive real zero and two negative real zeros
p. 85. integer; $a_0; a_n$
Example 4. $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12, \pm \frac{1}{3}; \pm \frac{2}{3}; \pm \frac{4}{3}$
p. 86. maximum number of real zeros; positive real zeros and negative real zeros; Rational Zeros Theorem; depressed equation
Example 5. $\left\{-3, -1, \frac{4}{3}\right\}$
Example 6. $\left\{-3, -\frac{3}{2}, -1\right\}$
Example 7. $\{-1, 2\}$
p. 87. linear; /or irreducible quadratic; one; third row; synthetic division; positive; zero; third row; synthetic; positive (or 0); and negative (or 0)
Example 8. (a) Every real zero of *f* lies between -5 and 5. **(b)** Every real zero of *f* lies between -5 and 5.
p. 88. $a < b; f(a); f(b); a$ and b **;** Answers will vary
Example 9. $f(-2)=-5; f(-1)=1;$ because $f(-2)<0$ and $f(-1)>0$, it follows from the Intermediate Value Theorem that the polynomial function *f* has at least one zero between -2 and -1.
p. 89. zero; 10; Intermediate Value Theorem; 3
Example 10. $\{\approx -1.32\}$

Section 4.4: Complex Zeros; Fundamental Theorem of Algebra

Exploration 1.
1. (a) $\{-2\}$ **(b)** $\{-3i, 3i\}$ **(c)** $\left\{1, -\frac{1}{2}+\frac{\sqrt{3}}{2}i, -\frac{1}{2}-\frac{\sqrt{3}}{2}i\right\}$
2. The number of zeros is equal to the degree of the polynomial.
3. If an imaginary number is a zero of a polynomial its conjugate will also be a zero of the polynomial.
p. 91. $f(x)=a_nx^n+a_{n-1}x^{n-1}+...+a_1x+a_0; a_n+a_{n-1}+...+a_1+a_0; a_n; f(r)=0$
p. 92. complex zero; $f(x)=a_n(x-r_1)(x-r_2)....(x-r_n); a_n, r_1, r_2, ..., r_n$
Example 1. five

p. 93. $\bar{r} = a - bi$; real; Because complex zeros occur as conjugate pairs in a polynomial function with real coefficients, there will always be an even number of zeros that are not real numbers. Consequently, since *f* is of odd degree, one of its zeros has to be a real number.

Example 2. $\{-5i, 1-i\}$

Example 3. $f(x) = x^4 + 6x^3 + 2x^2 - 26x + 17$

p. 93. linear factors; quadratic factors

Example 4. $\left\{-2, \frac{1}{3}, -3i, 3i\right\}$; $f(x) = (x+2)(3x-1)(x^2+9)$

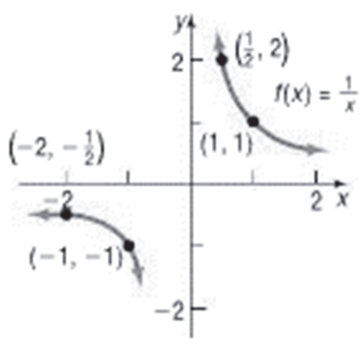
Example 5. $\left\{\frac{1}{2}, -1+2i, -1-2i\right\}$; $f(x) = (2x-1)(x^2+2x+5)$

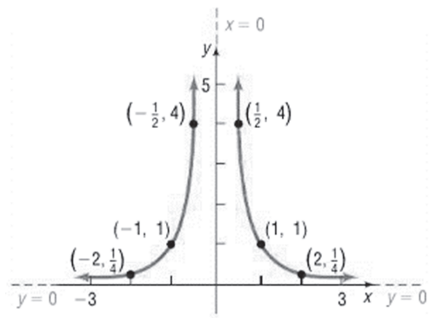
Section 4.5: Properties of Rational Functions

p. 95. $\frac{p(x)}{q(x)}$; zero polynomial; set of all real numbers; 0

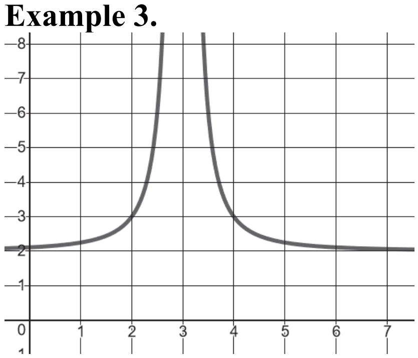
Example 1. (a) $\{x \mid x \neq -4\}$ **(b)** $\{x \mid x \neq -6, x \neq -2\}$ **(c)** $(-\infty, \infty)$ **(d)** $(-\infty, \infty)$ **(e)** $\{x \mid x \neq -2\}$

Example 2.





Example 3.



Exploration 1. 1(a) As $x \to \infty, f(x) \to 3$. **(b)** As $x \to -\infty, f(x) \to 3$. **(c)** Answers vary

p. 97. $x \to -\infty; x \to \infty; y = L; |R(x)| \to \infty; x = c;$ yes

(d) $y = 3$ **(d)** $x = -1$ **2(a))** As $x \to \infty, f(x) \to 0$. **(b)** As $x \to -\infty, f(x) \to 0$. **(c)** $y = 0$ **(d)** Answers vary **(e)** $x = 1$

(3)

Function	**Leading Term in the Numerator**	**Leading Term in the Denominator**	**Equation of the Horizontal Asymptote**	**Equation(s) of the Vertical Asymptote(s)**
$f(x)=\dfrac{3x+1}{x+1}$	$3x$	x	$y=3$	$x=-1$
$q(x)=\dfrac{x+2}{2x^2-2}$	x	$2x^2$	$y=0$	$x=1, x=-1$
$s(x)=\dfrac{2x^2+3x-2}{x^2+3x+2}$	$2x^2$	x^2	$y=2$	$x=-1$
$g(x)=\dfrac{-3x^2-2x+1}{x^3-1}$	$-3x^2$	x^3	$y=0$	$x=1$

(4) Answers will vary

p. 98. $x=r$; real zero; This means when the function is completely simplified.

Example 4. $x=-3; x=-2, x=2; x=-4, x=-1; x=2$

p. 99. $y=0; y=\dfrac{a_n}{b_m}; y=ax+b;$ no horizontal or oblique asymptotes

Example 5. Answers will vary

Example 6. $y=0; y=2; y=\dfrac{5}{2};$ no horizontal asymptote

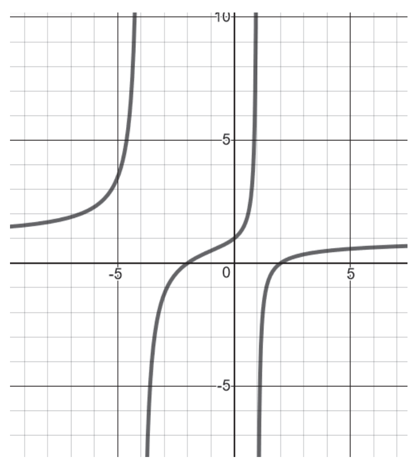
Example 7. $y=x+6$

Example 8. $y=2x-1$

Section 4.6: The Graph of a Rational Function

p. 101. numerator and denominator; domain of the rational function; lowest terms; intercepts; x-intercept; vertical asymptotes; opposite; same; horizontal; oblique; intersects

Example 1.



1. $R(x)=\dfrac{(x-2)(x+2)}{(x+4)(x-1)}; D:\{x \mid x\neq -4, x\neq 1\}$

3. $(0,1),(-2,0),(2,0);$ crosses at both zeros

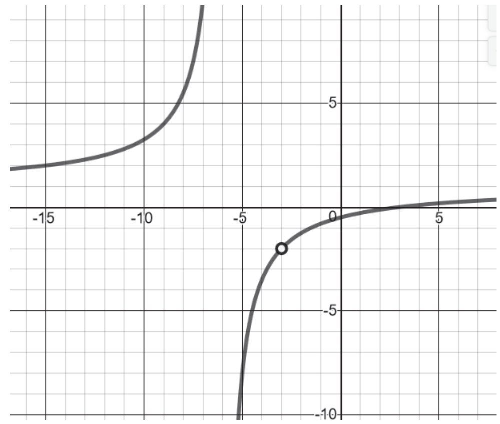
4. x = -4 and x = 1; opposite infinities at both VA

5. y = 1; never crosses

6. Above x-axis: $(-\infty,-4)\cup(-2,1)\cup(2,\infty);$ below x-axis: $(-4,-2)\cup(1,2)$

Example 2.

1. $R(x)=\dfrac{(x-3)(x+3)}{(x+3)(x+6)}; D:\{x \mid x\neq -6, x\neq -3\}$



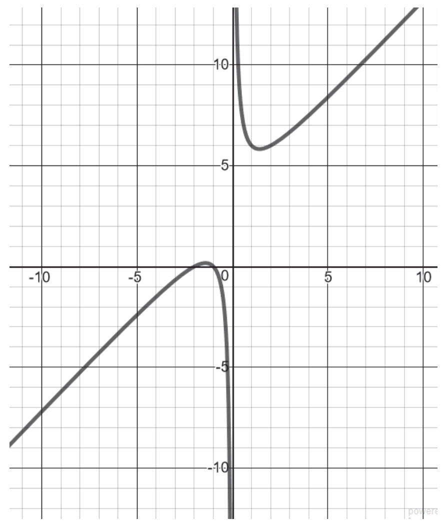
2. $R(x)=\dfrac{x-3}{x+6}$ **3.** $\left(0,-\dfrac{1}{2}\right),(3,0)$; crosses

4. x = -6; opposite infinity **5.** y = 1; never crosses

6. Above x-axis: $(-\infty,-6)\cup(3,\infty)$; below x-axis: $(-6,-3)\cup(-3,3)$

Example 3.

1. $R(x)=\dfrac{(x+1)(x+2)}{x}; D:\{x \mid x\neq 0\}$



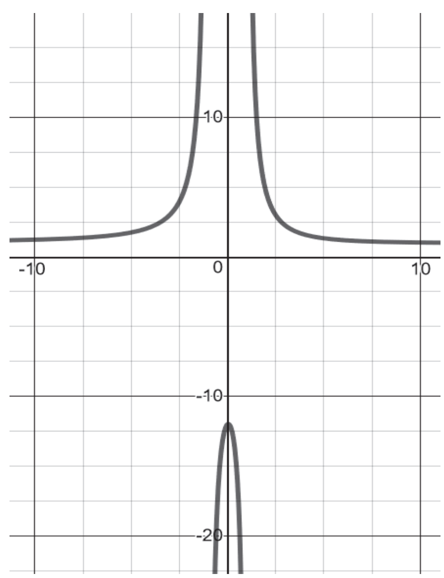
3. $(-2,0),(-1,0)$; crosses at both zeros

4. x = 0; opposite infinity **5.** $y = x + 3$; never crosses

6. Above x-axis: $(-2,-1)\cup(0,\infty)$; below x-axis: $(-\infty,-2)\cup(-1,0)$

Example 4.

1. $R(x)=\dfrac{x^2-x+12}{(x+1)(x-1)}; D:\{x \mid x\neq -1, x\neq 1\}$

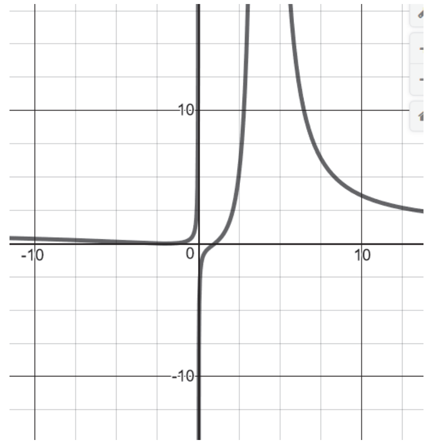


3. $(0,-12)$

4. x = 1; x = -1 opposite infinity **5.** y = 0; never crosses

6. Above x-axis: $(-\infty,-1)\cup(1,\infty)$; below x-axis: $(-1,1)$

Example 5.



1. $R(x)=\dfrac{(x-1)(x+2)^2}{x(x-4)^2}$; $D:\{x \mid x\neq 0, x\neq 4\}$

3. $(1,0),(-2,0)$; crosses at 1, touches at -2

4. $x = 0$; $x = 4$; opposite infinity at $x = 0$; same infinity at $x = 4$

5. $y = 1$; crosses at about -.2 and 1.7

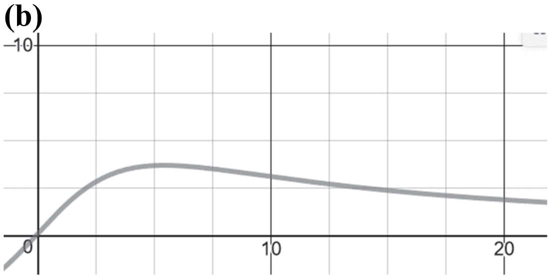
6. Above x-axis: $(-\infty,-2)\cup(-2,0)\cup(1,4)\cup(4,\infty)$

below x-axis: $(0,1)$

Example 6. $R(x)=\dfrac{-4(x+1)(x-9)}{(x+4)(x-4)}$

Example 7. (a) $t-axis; C(t)\to 0$

(b)



(c) about 5.4 minutes after the injection

Section 4.7: Polynomial and Rational Inequalities

Exploration 1. The solution to $f(x)>0$ are the x values that make the function positive. This is where the graph is above the x – axis; The solution to $f(x)\geq 0$ are the x values that make the function positive or equal to 0. These are the values where the graph is above or on the x – axis; The solution to $f(x)<0$ are the x values that make the function negative. This is where the graph is below the x – axis; The solution to $f(x)\leq 0$ are the x values that make the function negative or equal to 0. These are the values where the graph is below or on the x – axis;

Example 1. $\{x \mid x=-3, 1\leq x\leq 4\}$

Example 2. $\{x \mid -8\leq x\leq -2\}$

p. 107. left; $f(x)>0; f(x)\geq 0; f(x)<0; f(x)\leq 0$; single quotient; $f(x)=0$; undefined; $f(x)>0; f(x)<0$

Example 3. $\{x \mid x=-3, 1\leq x\leq 4\}$

Example 4. $\{x \mid -8\leq x\leq -2\}$

CHAPTER 5 Exponential and Logarithmic Equations

Section 5.1: Composite Functions

Exploration 1. (1) $g(h)=10h$ **(2)** $n(g)=0.8g$ **(3)** \$160 **(4)** $n(h)=8h$

p. 109. $f\circ g;(f\circ g)(x)=f(g(x))$

Example 1. (a) 53 **(b)** 501 **(c)** 245 **(d)** -107

Example 2.

x	$(f\circ g)(x)$
-2	7
-1	4
0	3
1	4
3	DNE

Example 3. (a) $32x^6+16x^3+5;D:(-\infty,\infty)$ **(b)** $32x^6+144x^4+216x^2+109;D:(-\infty,\infty)$

Example 4. $\{x\,|\,x\neq 1, x\neq 2\}$

Example 5. (a) $\dfrac{1}{\sqrt{x-1}};D:\{x\,|\,x>1\}$ **(b)** $x;D:\{x\,|\,x\neq 0\}$

Example 6.

$$\begin{aligned}(f\circ g)(x)&=f(g(x))\\&=f\left(\frac{1}{2}x\right)\\&=2\cdot\frac{1}{2}x\\&=x\end{aligned}$$

$$\begin{aligned}(g\circ f)(x)&=g(f(x))\\&=g(2x)\\&=\frac{1}{2}\cdot 2x\\&=x\end{aligned}$$

Example 7.

$$\begin{aligned}(f\circ g)(x)&=f(g(x))\\&=f(2x+1)\\&=\frac{1}{2}(2x+1-1)\\&=x\end{aligned}$$

$$\begin{aligned}(g\circ f)(x)&=g(f(x))\\&=g\left(\frac{1}{2}(x-1)\right)\\&=2\cdot\frac{1}{2}(x-1)+1\\&=x\end{aligned}$$

Example 8. $f(x)=x^4;g(x)=2x+3$

Example 9. $f(x)=\dfrac{1}{x};g(x)=2x^2-3$

Section 5.2: One-to-One Functions; Inverse Functions

p. 113. Two different outputs in the range; $f(x_1) \neq f(x_2)$

Example 1. (a) Not one-to-one because the output of "Pontiac" occurs for two different inputs.
(b) One-to-one because for each input there is one output and for each output there is one input.

p. 113. one point; Answers will vary

Example 2. Not one-to-one; one-to-one

Exploration 1. (1) 212; 100°C is equivalent to 212°F **(2)** $44.\overline{4}$; 112°F is equivalent to $44.\overline{4}$°C
(c) $C = \frac{5}{9}(F - 32)$

p. 115. exactly one; domain

Example 3. Domain: {Saturn, Pontiac, Honda, Chrysler}; Range: {Dan, John, Joe, Michelle}

Example 4. {(5,1), (8,2), (11,3), (14,4)}; Domain: {5,8,11,14}; Range: {1,2,3,4}

p. 115. Domain

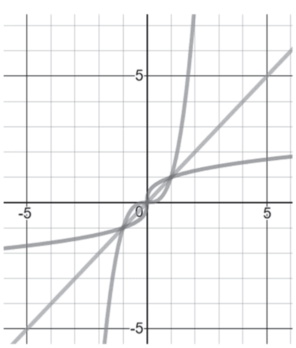
p. 116. x; x

Example 5.

$$\begin{aligned} g\left(g^{-1}(x)\right) &= g\left(\sqrt[3]{x-2}\right) \\ &= \left(\sqrt[3]{x-2}\right)^3 + 2 \\ &= x - 2 + 2 \\ &= x \end{aligned}$$

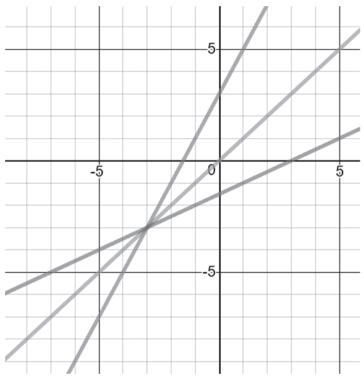
$$\begin{aligned} g^{-1}\left(g(x)\right) &= g^{-1}\left(x^3 + 2\right) \\ &= \sqrt[3]{x^3 + 2 - 2} \\ &= \sqrt[3]{x^3} \\ &= x \end{aligned}$$

Exploration 2. (1)



(2) The graphs of $y = x^3$ and its inverse $y = \sqrt[3]{x}$ are symmetric with respect to the line $y = x$

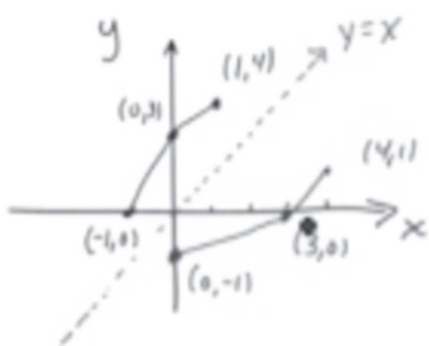
(3)



(4) The graphs of $y = 2x + 3$ and $y = \frac{1}{2}(x - 3)$ are symmetric with respect to the line $y = x$

p. 117. $y = x$

Example 6.



p. 117. $y = f^{-1}(x); f^{-1}(f(x)) = x$ and $f(f^{-1}(x)) = x$

Example 7. $f^{-1}(x) = \dfrac{x-2}{4}$

Example 8. $f^{-1}(x) = \dfrac{x+1}{2-x}$: Domain and range of *f*:
$D:(-\infty,-1)\cup(-1,\infty);R:(-\infty,2)\cup(2,\infty);$

Domain and range of $f^{-1}: D:(-\infty,2)\cup(2,\infty);R:(-\infty,-1)\cup(-1,\infty)$

Section 5.3: Exponential Functions

Example 1. (a) ≈2.639 **(b)** ≈2.657 **(c)** ≈2.665 **(d)** ≈2.665 **(e)** ≈2.665

p. 118.

$$a^s \cdot a^t = a^{s+t} \qquad (a^s)^t = a^{st} \qquad (ab)^s = a^s \cdot b^s$$

$$1^s = 1 \qquad a^{-s} = \frac{1}{a^s} = \left(\frac{1}{a}\right)^s \qquad a^0 = 1 \qquad \textbf{(1)}$$

Exploration 1.

# of Days you Go to Class	Pay ($)
0	10
1	20
2	40
3	80
4	160
5	320
6	640
7	1280
8	2560

(a) As the value of the independent variable increases by 1, the value of the dependent variable doubles. **(b)** $f(x)=10(2)^x$

p. 119. $f(x)=Ca^x$; the set of all real numbers; growth; initial value; If $a > 1$, a it is growth factor. If $a < 1$, it is a decay factor.

Exploration 2.

x	$f(x)=2^x$
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4
3	8

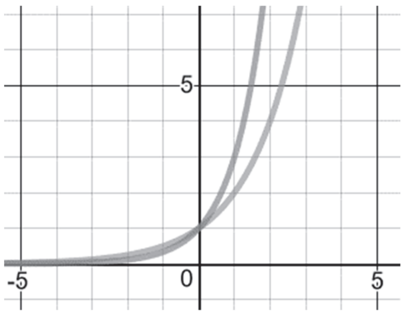
x	$g(x)=3x+2$
-2	-4
-1	-1
0	2
1	5
2	8
3	11

In *f*, for 1-unit changes in the input *x* of an exponential function, the **ratio** of consecutive outputs is constant. In *g*, for 1-unit changes in the input *x* of a linear function, the **difference** of consecutive outputs is constant.

p. 119. a; $af(x)$

Example 1. **(a)** Linear; $f(x)=1.5x-3$ **(b)** Exponential; $g(x)=16\left(\frac{1}{4}\right)^x$ **(c)** Linear; $h(x)=-4x+16$ **(d)** Exponential; $j(x)=3\left(\frac{3}{2}\right)^x$

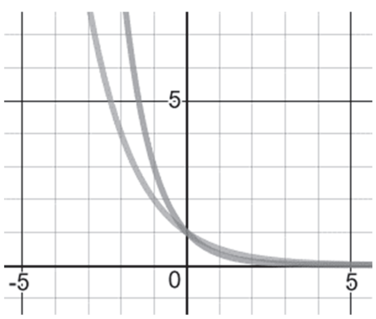
Exploration 3. (1)



(a) $D:(-\infty,\infty);R:(0,\infty)$ **(b)** No. $a^x \neq 0$ **(c)** No **(d)** There are no x-intercepts; y-intercept (0,1) **(e)** As x increases, the function's values are increasing. $y=3^x$ is increasing faster. **(f)** $y = 0$

p. 122. $(-\infty,\infty);(0,\infty)$; no; 1; $y = 0$; $x\to-\infty$ increasing; one-to-one; $\left(-1,\frac{1}{a}\right),(0,1),(1,a)$

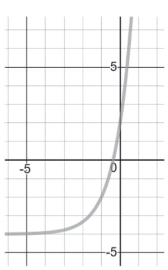
Exploration 3. (2)



(a) These functions decrease because the base is less than 1. **(b)** $D:(-\infty,\infty);R:(0,\infty)$ **(c)** $y = 0$

p. 123. $(-\infty,\infty);(0,\infty)$; no; 1; $y = 0$; $x\to\infty$; decreasing; one-to-one; $\left(-1,\frac{1}{a}\right),(0,1),(1,a)$

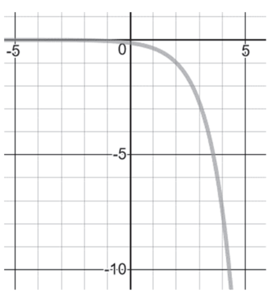
Example 2. $D:(-\infty,\infty);R:(-4,\infty);HA:y=-4$



Exploration 4.

n	$f(n)=\left(1+\dfrac{1}{n}\right)^n$
10	2.5937
50	2.6916
100	2.7048
500	2.7156
1000	2.7169
10,000	2.7181
100,000	2.718268
1,000,000	2.7182805

Example 3. $D:(-\infty,\infty);R:(-\infty,0);HA:y=0$

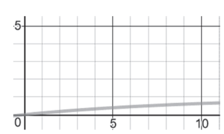


p. 125. The variable is in the exponent.

p. 126. $u=v$

Example 4. **(a)** $\{2\}$ **(b)** $\{-6\}$ **(c)** $\left\{\dfrac{3}{2}\right\}$ **(d)** $\left\{\dfrac{3}{2}\right\}$ **(e)** $\{2,4\}$ **(f)** $\left\{-1,-\dfrac{1}{3}\right\}$ **(g)** $\left\{\dfrac{1}{9}\right\}$ **(h)** $\{-5,1\}$

Example 5. **(a)** 0.63 **(b)** 0.98 **(c)** As $t\to\infty, F(t)\to 1$ **(d)**



(e) ≈ 7 minutes

Section 5.4: Logarithmic Functions

Exploration 1. **(a)** 3 **(b)** 2 **(c)** 8 **(d)** 4 **(e)** 16 **(f)** $\dfrac{1}{2}$

p. 128. $y=\log_a x$ if and only if $x=a^y; x>0$

Example 1. **(a)** $\log_5 t=8$ **(b)** $\log_x 12=-2$ **(c)** $\log_e 10=x$

Example 2. **(a)** $2^y=21$ **(b)** $z^6=12$ **(c)** $2^a=10$

Example 3. **(a)** 4 **(b)** -3 **(c)** 0 **(d)** 4 **(e)** 2 **(f)** $\dfrac{1}{2}$ **(g)** -3 **(h)** 4

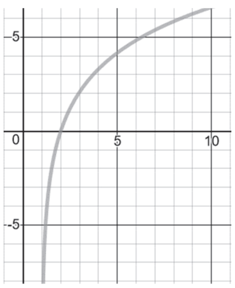
p. 129. Domain: all Real Numbers greater than 0; Range: all real numbers

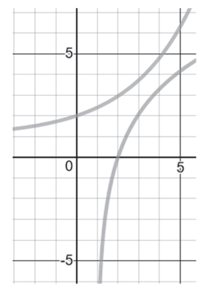
p. 130. $0 < x < \infty; -\infty < y < \infty$

Example 4. (a) $(2,\infty)$ **(b)** $(-\infty,-3)\cup(1,\infty)$ **(c)** $\{x \mid x \neq 1\}$ **(d)** $\{x \mid x \neq 0\}$

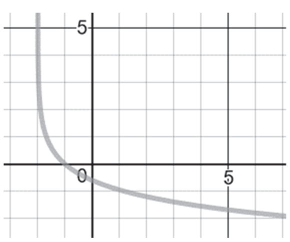
p. 130. $(0,\infty);(-\infty,\infty)$; 1; no; vertical; $0 < a < 1; a > 1$; $(1,0),(a,1),\left(\frac{1}{a},-1\right)$; smooth and continuous; corners or gaps

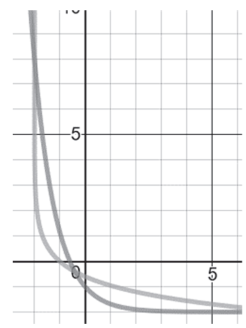
Example 5. (a) **(b)** $(1,\infty)$ **(c)** $(-\infty,\infty); x = 1$ **(d)** $f^{-1}(x) = e^{\frac{x}{3}} + 1$ **(e)** $D:(-\infty,\infty); R:(1,\infty)$ **(f)**





Example 6. (a) **(b)** $(-2,\infty)$ **(c)** $(-\infty,\infty); x = -2$ **(d)** $f^{-1}(x) = 10^{-\frac{x}{2}} - 2$ **(e)** $D:(-\infty,\infty); R:(-2,\infty)$ **(f)**





Example 7. (a) $\left\{\frac{7}{2}\right\}$ **(b)** $\{7\}$ **(c)** $\{1000\}$ **(d)** $\{e^2\}$ **(e)** $\left\{\frac{1}{24}\right\}$ **(f)** $\left\{-\frac{1}{5}\right\}$

Example 8. (a) $\{\ln 7\}$ **(b)** $\left\{\frac{\ln 3}{3}\right\}$ **(c)** $\left\{\frac{\ln(9)+1}{5}\right\}$ **(d)** $\left\{\frac{\log 5}{2}\right\}$ **(e)** $\left\{\frac{\ln(4)-1}{2}\right\}$

Section 5.5: Properties of Logarithms

Exploration 1. (a) $\{0\}$ **(b)** $\{0\}$ **(c)** $\{0\}$ **(d)** $\{0\}$ **(e)** $\{1\}$ **(f)** $\{1\}$ **(g)** $\{1\}$ **(h)** $\{1\}$

p. 135. 0; 1

Exploration 2. (a) x **(b)** x

p. 135. r; M

Exploration 3.

(a)

$$\log(100\cdot 10) = \log(100) + \log(10)$$
$$\log(1000) = 2 + 1$$
$$3 = 3$$

(b)

$$\log\left(\frac{1000}{100}\right) = \log(1000) - \log(100)$$
$$\log(10) = 3 - 2$$
$$1 = 1$$

(c)

$$\log 10^3 = 3\log(10)$$
$$\log(1000) = 3(1)$$
$$3 = 3$$

p. 136. $\log_a(MN) = \log_a(M) + \log_a(N); \log_a\left(\frac{M}{N}\right) = \log_a(M) - \log_a(N); \log_a M^r = r\log_a M$

Example 1. (a) 18 **(b)** -5 **(c)** 20 **(d)** 3

Example 2. (a) 1 **(b)** $-\frac{4}{3}$ **(c)** 15 **(d)** 5

Example 3. (a) $\log_3(x-1) + 2\log_3(x+2)$ **(b)** $2\log_5 x + 3\log_5 y - \frac{1}{2}\log_5 z$

Example 4. (a) $\log_2[x(x-3)]$ **(b)** $\log_6\left(\frac{z^3}{y^2}\right)$ **(c)** $\ln\left[\frac{(x-2)\sqrt{x}}{(x+3)^5}\right]$

p. 137. $\log_a M = \log_a N; M = N; \frac{\log_b M}{\log_b a}$; The change of base formula allows us to change the base of the logarithm to either base 10 or e so that we can use a calculator to evaluate the logarithm.

Example 5. (a) ≈ 2.2619 **(b)** ≈ 2.9723

$\log_a 1 = 0$	$\log_a a = 1$	$\log_a M^r = r\log_a M$
$a^{\log_a M} = M$	$\log_a a^r = r$	$a^r = e^{r\ln a}$
$\log_a(MN) = \log_a M + \log_a N$		$\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$
If $M = N$, then $\log_a M = \log_a N$.		If $\log_a M = \log_a N$, then $M = N$.
$\log_a M = \frac{\log_b M}{\log_b a}$		

Section 5.6: Logarithmic and Exponential Equations

Example 1. (a) $\{2\}$ **(b)** $\{-1, 0\}$

Example 2. **(a)** $\left\{1+\sqrt{3}\right\}$ **(b)** $\{1\}$ **(c)** $\left\{-\dfrac{9}{11}\right\}$ **(d)** $\left\{\dfrac{3e^2+1}{e^2-3}\right\}$

Example 3. **(a)** $\{1\}$ **(b)** $\left\{\dfrac{\ln 7}{\ln 3}\right\}$ **(c)** $\left\{\dfrac{\ln 3-\ln 5}{\ln 2}\right\}$ **(d)** $\left\{\dfrac{3\log 5+\log 2}{\log 2-2\log 5}\right\}$

Example 4. $\{\approx -0.567\}$

Section 5.7: Financial Models

p. 142. 1; 12; 2; 365; 4

Example 1. \$1200

p. 142. At the end of 6 months: \$10,600; At the end of 12 months: \$11,236; In general, at the end of 6 months = $10,000(1.06)$ and at the end of 12 months = $10,000(1.06)^2 = 10,000\left(1+\dfrac{.12}{2}\right)^2$

p. 143. $A = P\left(1+\dfrac{r}{n}\right)^{nt}$

Example 2. **(a)** \$1100 **(b)** \$1103.81 **(c)** \$1104.71 **(d)** \$1105.16 **(e)** As *n* increases, *A* increases

Example 3. **(a)** \$2 **(b)** \$2.44 **(c)** \$2.61 **(d)** \$2.71 **(e)** As *n* increases, *A* increases. Also, as $n\to\infty, \left(1+\dfrac{r}{n}\right)^n \to e \approx 2.71$

p. 144. $n\to\infty, \left(1+\dfrac{r}{n}\right)^n \to e\approx 2.71; A = Pe^{rt}$

Example 4. \$2166.57

Example 5. \$1042.82; 4.19%

p. 144. $\left(1+\dfrac{r}{n}\right)^n - 1; e^r - 1$

Example 6. 5.09%

Example 7. Bank B

p. 145. $A\cdot\left(1+\dfrac{r}{n}\right)^{-nt}; Ae^{-rt}$

Example 8. \$88.72

Example 9. **(a)** ≈ 8.69 years **(b)** ≈ 8.66 years

Example 10. $\approx 11.7\%$

Section 5.8: Exponential Growth and Decay Models; Newton's Law; Logistic Growth and Decay Models

p. 147. $N_0 e^{kt}$; the initial number of cells; positive

Example 1. (a) 100 grams **(b)** 4.5% **(c)** 125.2 grams **(d)** $\approx$7.5 days **(e)** $\approx$15.4 days

Example 2. (a) $\approx 278{,}576$ bacteria **(b)** ≈ 28.2 hours

p. 148. $A_0 e^{kt}$; the original amount of radioactive material; negative

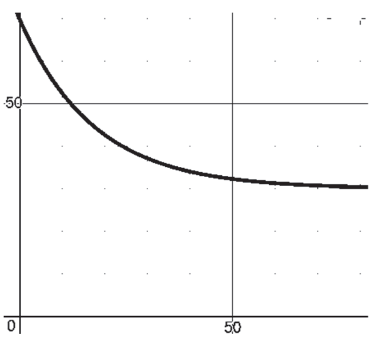
Example 3. (a) 8.7% decay **(b)** $\approx$129.453 grams **(c)** $\approx$7.97 days

Example 4. $\approx$33,000 years

p. 149. $T+(u_0-T)e^{kt}$; constant temperature; initial temperature; negative

Example 5. (a) $\approx$12 minutes

(b)



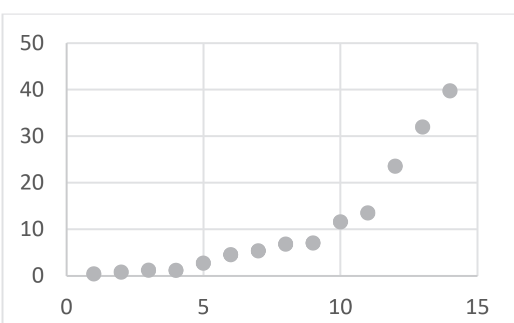
(d) $\approx$36 minutes **(e)** As $t \to \infty, u(t) \to 30$

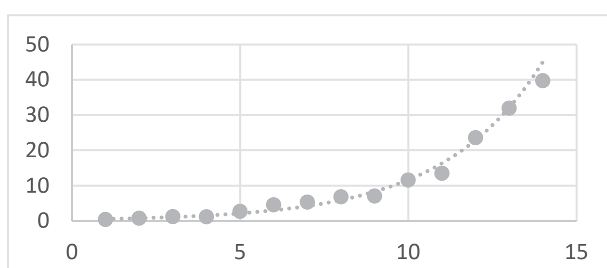
p. 150. $\dfrac{c}{1+ae^{-bt}}; b>0; b<0; (-\infty,\infty); (0,c); P(0); y=0, y=c; b>0; b<0; \dfrac{1}{2}$

Example 6. (a) -0.0581 **(b)** 95 **(c)** 59.6 years **(d)** If you evaluate the function at $t = 0$ you should get around 100% of the wood remaining which is close to the number in the numerator.

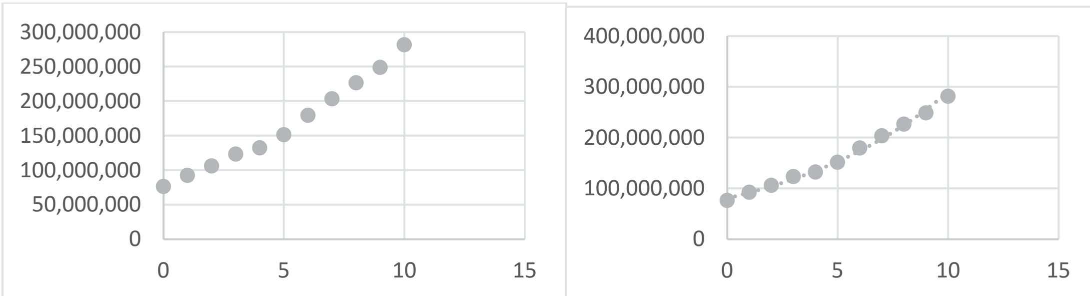
Section 5.9: Building Exponential, Logarithmic, and Logistic Models from Data

Example 1. (a) **(b)** $y=0.394503(1.40276)^x$ **(c)** $A(t)=0.394503e^{0.33844t}$ **(e)** $\approx$63.21





Example 2. (a) **(b)** $y = \dfrac{799475916.5}{1+1563441\times10^{14}e^{-0.0160338024x}}$ **(d)** 799,475,916.5 **(e)** 292,251,183



(f) about 2007

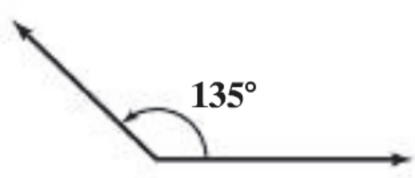
CHAPTER 6: Trigonometric Functions

Section 6.1: Angles, Arc Length, and Circular Motion

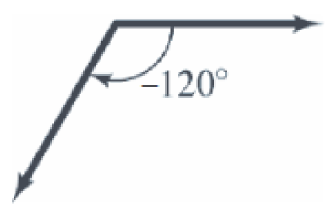
p. 153. ray; vertex; angle; initial; terminal; counterclockwise; clockwise
Alpha; Beta; Gamma
standard position
quadrantal

Exploration 1. counterclockwise; $\dfrac{90}{360}$ or $\dfrac{1}{4}$; 90; $\dfrac{180}{360}$ or $\dfrac{1}{2}$; 180

Example 1. (a)



(b)



p. 154. 1′; 1″

Example 2. (a) 45.1708° **(b)** 34.2575° **(c)** 12°52′12″ **(d)** 1°31′48″

p. 155. $\dfrac{s}{r}$; θr

Example 3. 2 meters

Example 4. 2.5

Exploration 2. 2πr; 2π; 360°; 2π; 360°; 180°; π; $\dfrac{\pi}{180} \approx 0.017$; $\dfrac{180}{\pi} \approx 57.3$

Example 5. (a) $\dfrac{4\pi}{9} \approx 1.40$ **(b)** $\dfrac{7\pi}{9} \approx 2.44$ **(c)** $-\dfrac{\pi}{6} \approx -0.52$ **(d)** $\dfrac{5\pi}{9} \approx 1.75$

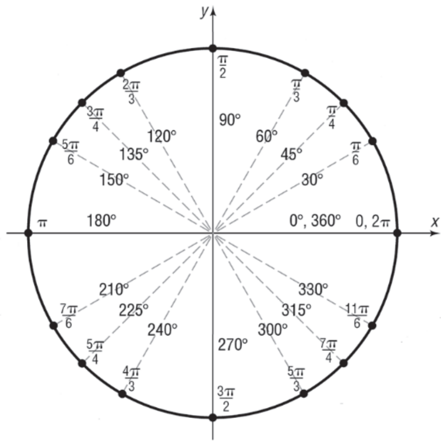
Example 6. (a) -108° **(b)** 60° **(c)** 480° **(d)** $\dfrac{360°}{\pi} \simeq 114.59°$

Example 7. ≈143.24°

Example 8.

θ (degrees)	0°	30°	45°	60°	90°	180°	270°	360°
θ (radians)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
θ (approx. radians)	0	0.52	0.79	1.05	1.57	3.14	4.71	6.28

Example 9.



Exploration 3. 360°; $\frac{1}{2}r^2\theta$

Example 10. 8.73 ft^2

Exploration 4. rate; time; $\frac{s}{t}$; $\frac{s}{t}$; $\frac{\theta}{t}$; $\frac{s}{t}$; $\frac{\theta}{t}$

Example 11. $v = r\omega$; 794.168 mph

Section 6.2: Trigonometric Approach: Unit Circle Approach

Exploration 1. (-1, 0); (0, -1); (0, 1); (1, 0); (1, 0); (0, -1)
y; x;
0; -1; 0; 1
-1; 0; 0; 1
1; 0; -1; 0

p. 160. $\frac{y}{x}; \frac{1}{y}; \frac{1}{x}; \frac{x}{y}$

Example 1. $\sin t = \frac{2\sqrt{6}}{5}; \cos t = -\frac{1}{5}; \tan t = -2\sqrt{6}; \csc t = \frac{5\sqrt{6}}{12}; \sec t = -5; \cot t = -\frac{\sqrt{6}}{12}$

Example 2.

θ (Radians)	θ (Degrees)	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
0	0°	0	1	0	Not defined	1	Not defined
$\frac{\pi}{2}$	90°	1	0	Not defined	1	Not defined	0
π	180°	0	-1	0	Not defined	-1	Not defined
$\frac{3\pi}{2}$	270°	-1	0	Not defined	-1	Not defined	0

Exploration 2. $\frac{\pi}{4} = 45°$; 1; $\sqrt{2}$; $\sin\frac{\pi}{4} = \sin 45° = \frac{\sqrt{2}}{2}; \cos\frac{\pi}{4} = \cos 45° = \frac{\sqrt{2}}{2}$;

$\tan\frac{\pi}{4} = \tan 45° = 1; \cot\frac{\pi}{4} = \cot 45° = 1$; $\sec\frac{\pi}{4} = \sec 45° = \sqrt{2}; \csc\frac{\pi}{4} = \csc 45° = \sqrt{2}$

Exploration 3. 1; $\sqrt{3}$; $\sin\frac{\pi}{6} = \sin 30° = \frac{1}{2}; \cos\frac{\pi}{3} = \cos 60° = \frac{1}{2}$;

$\cos\frac{\pi}{6} = \cos 30° = \frac{\sqrt{3}}{2}; \sin\frac{\pi}{3} = \sin 60° = \frac{\sqrt{3}}{2}$; $\tan\frac{\pi}{6} = \tan 30° = \frac{\sqrt{3}}{3}; \cot\frac{\pi}{3} = \cot 60° = \frac{\sqrt{3}}{3}$;

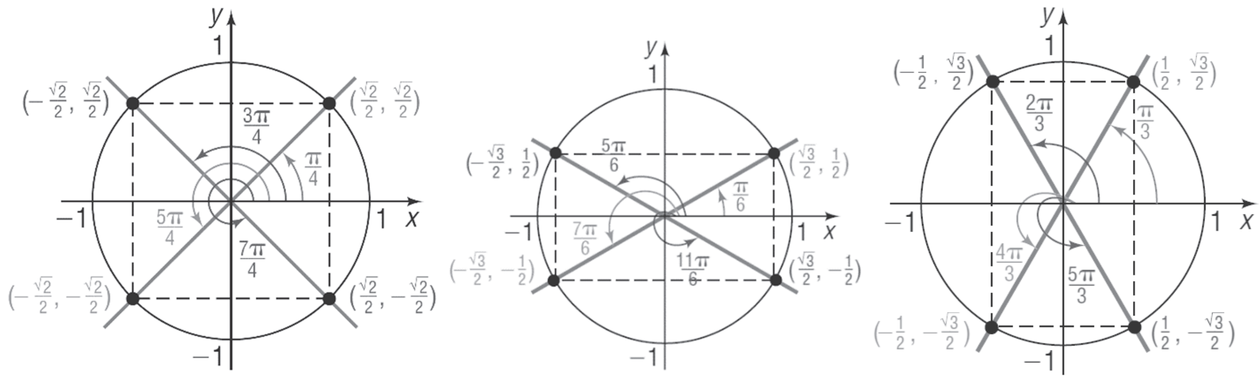
$\csc\frac{\pi}{6} = \csc 30° = 2; \sec\frac{\pi}{3} = \sec 60° = 2$; $\sec\frac{\pi}{6} = \sec 30° = \frac{2\sqrt{3}}{3}; \csc\frac{\pi}{3} = \csc 60° = \frac{\sqrt{3}}{3}$;

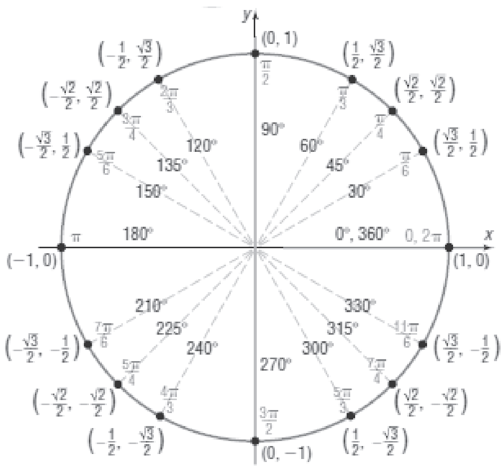
$\cot\frac{\pi}{6} = \cot 30° = \sqrt{3}; \tan\frac{\pi}{3} = \tan 60° = \sqrt{3}$

p. 163.

θ (Radians)	θ (Degrees)	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
$\frac{\pi}{6}$	30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
$\frac{\pi}{4}$	45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
$\frac{\pi}{3}$	60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$

Exploration 4.





Example 3. $0; -1; -\frac{\sqrt{2}}{2}+1; \frac{1}{2}; -2$

Example 4. 0.97; 0.27; -1.19

p. 165.

$\sin\theta = \frac{y}{r}$	$\cos\theta = \frac{x}{r}$	$\tan\theta = \frac{y}{x} \quad x \neq 0$
$\csc\theta = \frac{r}{y} \quad y \neq 0$	$\sec\theta = \frac{r}{x} \quad x \neq 0$	$\cot\theta = \frac{x}{y} \quad y \neq 0$

Example 5. $\sin\theta = -\frac{4}{5}; \cos\theta = \frac{3}{5}; \tan\theta = -\frac{4}{3}; \csc\theta = -\frac{5}{4}; \sec\theta = \frac{5}{3}; \cot\theta = -\frac{3}{4}$

Section 6.3: Properties of the Trigonometric Functions

Exploration 1. For the sine function, there is no concern about dividing by 0, so the domain can be any angle. Since the unit circle contains values no greater than 1 and no less than -1, the range of the sine function is [-1, 1]

θ; x; all real numbers; [-1, 1]

For the tangent function, the range consist of $\frac{y}{x}$ where both numerator and denominator are [-1, 1]. Therefore, the output gets infinitely large or small and the range is all real numbers.

θ; $\frac{x}{y}$; all real numbers, except integer multiples of π or 180°; all real numbers

For the cosecant function, the range consist of $\frac{1}{y}$ where the denominator is [-1, 1]. Therefore, the output is all real numbers less than or equal to -1 or greater than or equal to 1.

θ; $\frac{1}{x}$; all real numbers, except odd integer multiples of $\frac{\pi}{2}$ or 90°; all real numbers less than or equal to -1 or greater than or equal to 1.

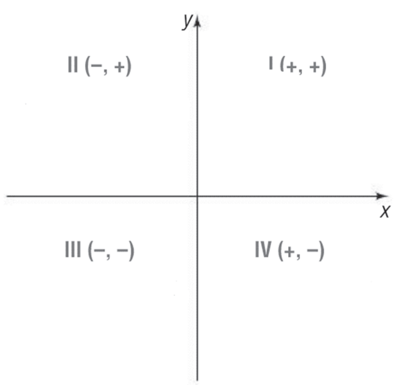
Exploration 2. A full rotation around a circle is 360°, so 420° represents one full rotation plus 60°. Therefore 420° and 60° represent the same location on a circle, and therefore produce the same cosine.

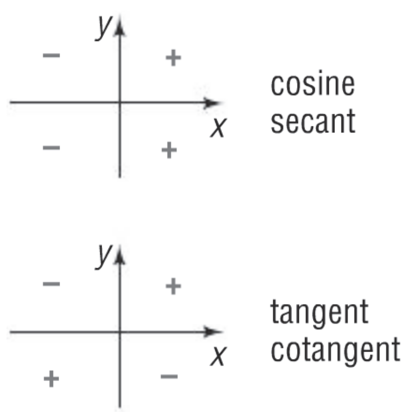
p. 167. 2π; $\sin\theta$; 2π; $\cos\theta$

$f(\theta)$; 2π, 2π; π; 2π, 2π; π

Example 1. (a) $\frac{1}{2}$ **(b)** -1

Exploration 3.





Example 2. Quadrant II

Example 3. $\tan\theta = \frac{1}{3}; \csc\theta = \sqrt{10}; \sec\theta = \frac{\sqrt{10}}{3}; \cot\theta = 3$

Exploration 4. $\cos\theta; \sin\theta$; Pythagorean Identity; Divide both sides by $\sin^2\theta$; Divide both sides by $\cos^2\theta$

Example 4. (a) 1 **(b)** 0

Example 5. Quadrant III; $\cos\theta = -\frac{\sqrt{5}}{3}; \tan\theta = \frac{2\sqrt{5}}{5}; \csc\theta = -\frac{3}{2}; \sec\theta = -\frac{3\sqrt{5}}{5}; \cot\theta = \frac{\sqrt{5}}{2}$

Exploration 5. $f(x) = f(-x); -f(x) = f(-x)$; sine functions are odd because $\sin(\theta) = y$ and $\sin(-\theta) = -y$; cosine functions are even because $\cos(\theta) = x$ and $\cos(-\theta) = x$

$$\sin(-\theta) = -\sin\theta \qquad \cos(-\theta) = \cos\theta \qquad \tan(-\theta) = -\tan\theta$$

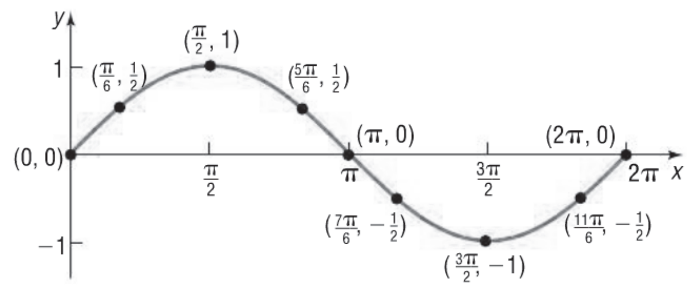
$$\csc(-\theta) = -\csc\theta \qquad \sec(-\theta) = \sec\theta \qquad \cot(-\theta) = -\cot\theta$$

Example 6. (a) $\frac{1}{2}$ **(b)** $-\frac{1}{2}$ **(c)** -1

Section 6.4: Graphs of the Sine and Cosine Functions

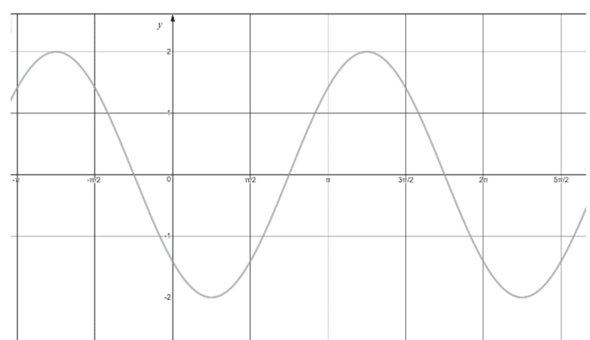
Exploration 1.

x	$y = \sin x$	(x, y)
0	0	$(0, 0)$
$\frac{\pi}{6}$	$\frac{1}{2}$	$\left(\frac{\pi}{6}, \frac{1}{2}\right)$
$\frac{\pi}{2}$	1	$\left(\frac{\pi}{2}, 1\right)$
$\frac{5\pi}{6}$	$\frac{1}{2}$	$\left(\frac{5\pi}{6}, \frac{1}{2}\right)$
π	0	$(\pi, 0)$
$\frac{7\pi}{6}$	$-\frac{1}{2}$	$\left(\frac{7\pi}{6}, -\frac{1}{2}\right)$
$\frac{3\pi}{2}$	-1	$\left(\frac{3\pi}{2}, -1\right)$
$\frac{11\pi}{6}$	$-\frac{1}{2}$	$\left(\frac{11\pi}{6}, -\frac{1}{2}\right)$
2π	0	$(2\pi, 0)$



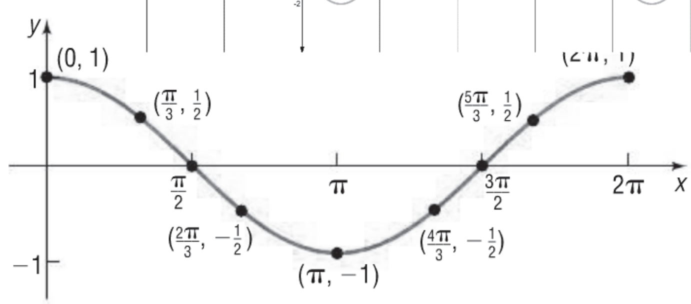
p. 172. the set of all real numbers; all real numbers from -1 to 1, inclusive; odd function; the origin; 2π; …, -2π, -π, 0, π, 2π, 3π,…; 0; 1; $\ldots, -\frac{3\pi}{2}, \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \ldots$; -1; $\ldots, -\frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}, \ldots$

Example 1. (a) $y = \sin x$ **(b)** reflect about the x – axis; horizontal compression by a factor of $\frac{1}{2}$; shifted left $\frac{\pi}{4}$ units



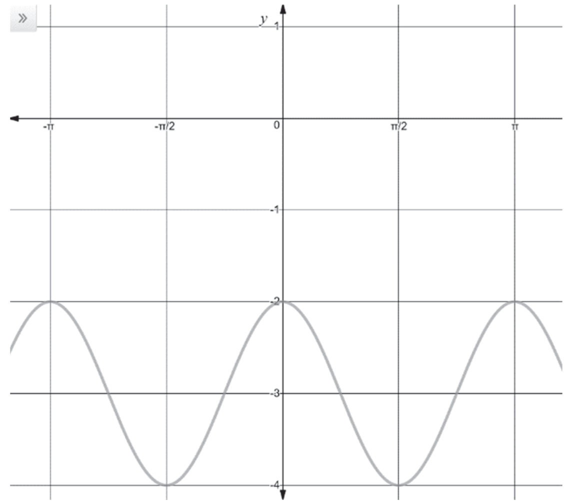
Exploration 2.

x	$y = \cos x$	(x, y)
0	1	$(0, 1)$
$\frac{\pi}{3}$	$\frac{1}{2}$	$\left(\frac{\pi}{3}, \frac{1}{2}\right)$
$\frac{\pi}{2}$	0	$\left(\frac{\pi}{2}, 0\right)$
$\frac{2\pi}{3}$	$-\frac{1}{2}$	$\left(\frac{2\pi}{3}, -\frac{1}{2}\right)$
π	-1	$(\pi, -1)$
$\frac{4\pi}{3}$	$-\frac{1}{2}$	$\left(\frac{4\pi}{3}, -\frac{1}{2}\right)$
$\frac{3\pi}{2}$	0	$\left(\frac{3\pi}{2}, 0\right)$
$\frac{5\pi}{3}$	$\frac{1}{2}$	$\left(\frac{5\pi}{3}, \frac{1}{2}\right)$
2π	1	$(2\pi, 1)$

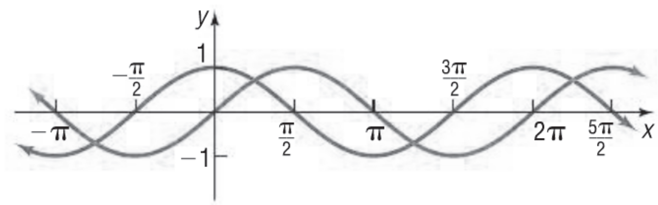


p. 174. the set of all real numbers; all real numbers from -1 to 1, inclusive; even function; the y - axis; 2π; $\ldots, -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$, 1; 1; …, -2π, 0, π, 2π, 4π, 6π,…; -1; …, -π, π, 3π, 5π,…

Example 2. (a) $y = \cos x$ **(b)** horizontal compression by a factor of $\frac{1}{2}$; shifted down 3 units



Exploration3.



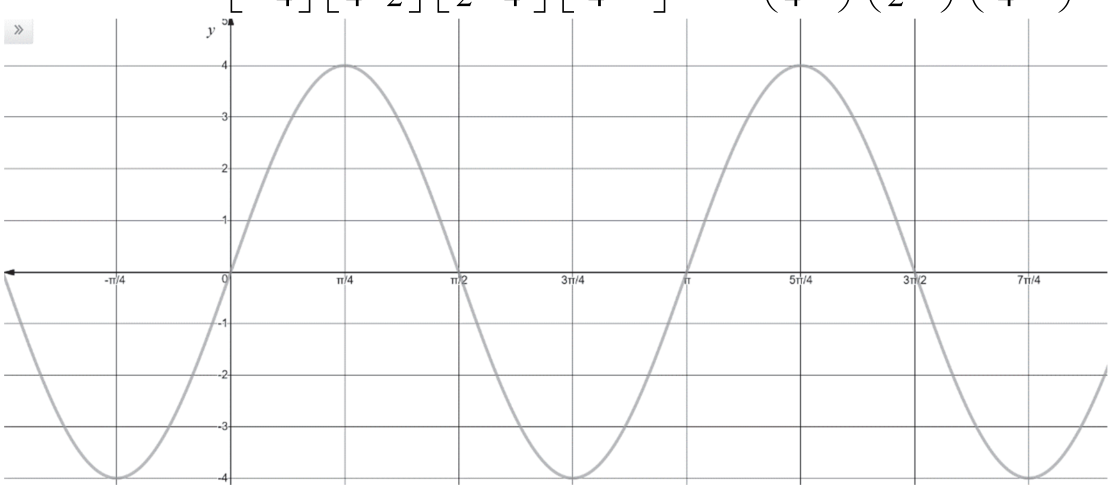
The graph is the same as the sine function; $\sin(x)$

Exploration 4. [-1, 1]; [-2, 2]; $\left[-\frac{1}{2}, \frac{1}{2}\right]$; $-|A|$; $|A|$; $-|A|$; $|A|$; The absolute value makes the amplitude positive, which is appropriate for a height; $2\pi; \frac{2\pi}{3}; 8\pi$; $\frac{2\pi}{\omega}$; the change represents a horizontal compression or stretch by a factor of $\frac{1}{\omega}$.

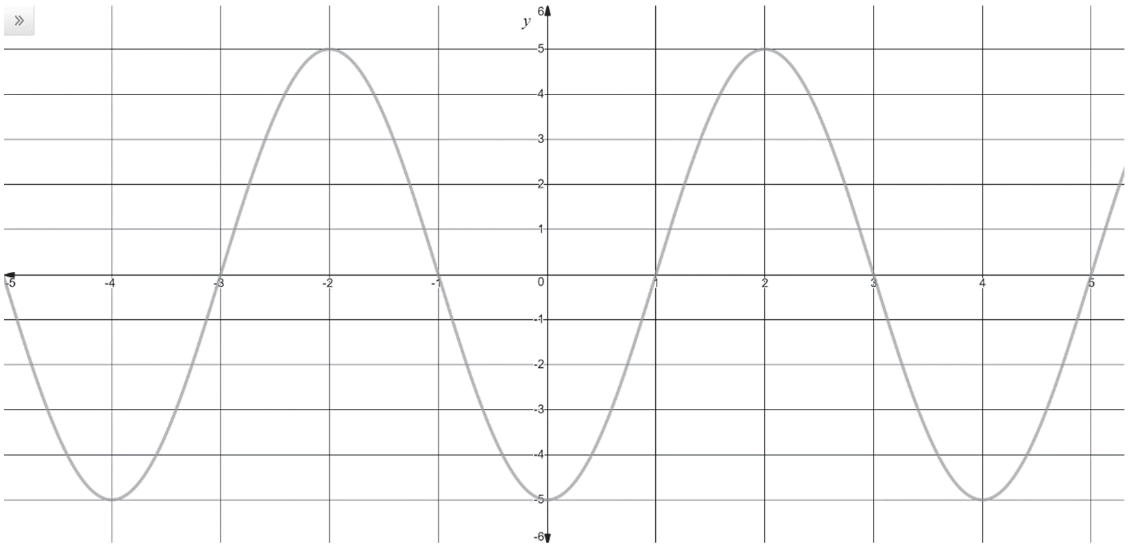
p. 175. $\omega > 0; |A|; T; \frac{2\pi}{\omega}$

Example 3. (a) $4, \frac{2\pi}{3}$ **(b)** $\frac{1}{2}; 2$

Example 4. $4; \pi; \left[0, \frac{\pi}{4}\right], \left[\frac{\pi}{4}, \frac{\pi}{2}\right], \left[\frac{\pi}{2}, \frac{3\pi}{4}\right], \left[\frac{3\pi}{4}, \pi\right]; (0,0), \left(\frac{\pi}{4}, 4\right), \left(\frac{\pi}{2}, 0\right), \left(\frac{3\pi}{4}, 4\right), (\pi, 0);$



Example 5. $5;4;[0,1],[1,2],[2,3],[3,4];(0,-5),(1,0),(2,5),(3,0),(4,-5);$



p. 177.

Steps for Graphing a Sinusoidal Function of the Form $y = A\sin(\omega x)$ or $y = A\cos(\omega x)$ Using Key Points

STEP 1: Determine the amplitude and period of the sinusoidal function.

STEP 2: Divide the interval $\left[0, \frac{2\pi}{\omega}\right]$ into four subintervals of the same length.

STEP 3: Use the endpoints of these subintervals to obtain five key points on the graph.

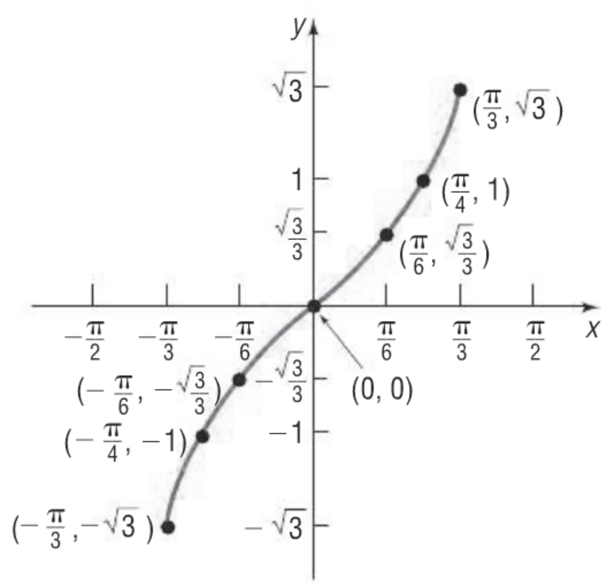
STEP 4: Plot the five key points, and draw a sinusoidal graph to obtain the graph of one cycle. Extend the graph in each direction to make it complete.

Example 6. $y = -3\cos\left(\frac{\pi}{4}x\right)$

Section 6.5: Graphs of the Tangent, Cotangent, Cosecant, and Secant Functions

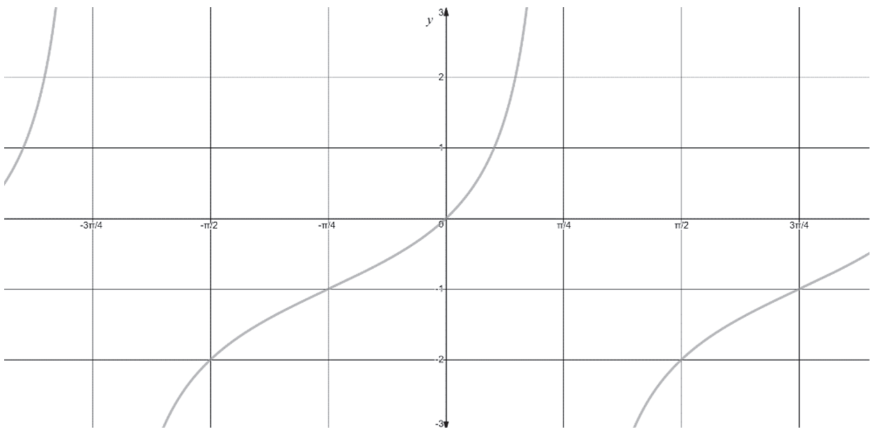
Exploration 1. π; repeats; vertical asymptote

x	$y = \tan x$	(x, y)
$-\frac{\pi}{3}$	$-\sqrt{3} \approx -1.73$	$\left(-\frac{\pi}{3}, -\sqrt{3}\right)$
$-\frac{\pi}{4}$	-1	$\left(-\frac{\pi}{4}, -1\right)$
$-\frac{\pi}{6}$	$-\frac{\sqrt{3}}{3} \approx -0.58$	$\left(-\frac{\pi}{6}, -\frac{\sqrt{3}}{3}\right)$
0	0	$(0, 0)$
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{3} \approx 0.58$	$\left(\frac{\pi}{6}, \frac{\sqrt{3}}{3}\right)$
$\frac{\pi}{4}$	1	$\left(\frac{\pi}{4}, 1\right)$
$\frac{\pi}{3}$	$\sqrt{3} \approx 1.73$	$\left(\frac{\pi}{3}, \sqrt{3}\right)$



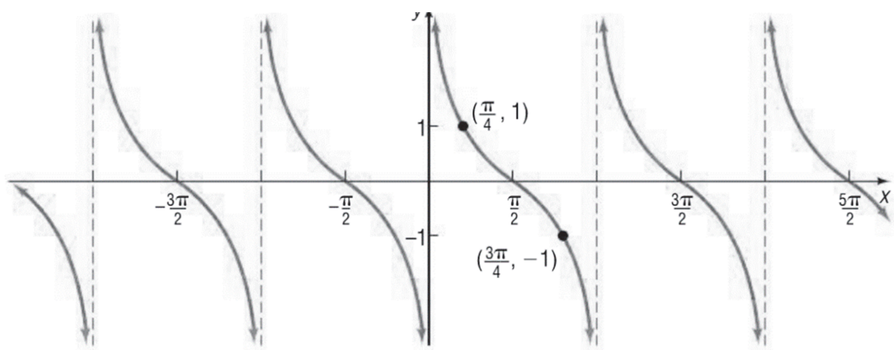
p. 178. the set of all real numbers, except odd multiples of $\frac{\pi}{2}$; the set of all real numbers; odd function; the origin; π; ; …, -2π, -π, 0, π, 2π, 3π,…; 0; $\ldots, -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$

Example 1. (a) $y = \tan x$ **(b)** shifted left $\frac{1}{4}$ unit; shifted down 1 unit



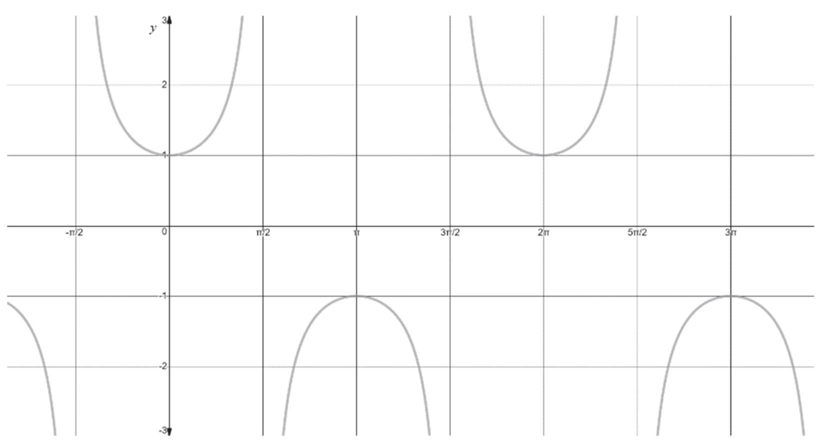
Exploration 2. π; repeats

x	$y = \cot x$	(x, y)
$\frac{\pi}{6}$	$\sqrt{3}$	$\left(\frac{\pi}{6}, \sqrt{3}\right)$
$\frac{\pi}{4}$	1	$\left(\frac{\pi}{4}, 1\right)$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{3}$	$\left(\frac{\pi}{3}, \frac{\sqrt{3}}{3}\right)$
$\frac{\pi}{2}$	0	$\left(\frac{\pi}{2}, 0\right)$
$\frac{2\pi}{3}$	$-\frac{\sqrt{3}}{3}$	$\left(\frac{2\pi}{3}, -\frac{\sqrt{3}}{3}\right)$
$\frac{3\pi}{4}$	-1	$\left(\frac{3\pi}{4}, -1\right)$
$\frac{5\pi}{6}$	$-\sqrt{3}$	$\left(\frac{5\pi}{6}, -\sqrt{3}\right)$



Exploration 3. $\csc x = \dfrac{1}{\sin x}; \sec x = \dfrac{1}{\cos x}; \left(\dfrac{\pi}{6}, 2\right)$; vertical asymptote; $\left(\dfrac{\pi}{3}, 2\right)$; vertical asymptote

Example 2.

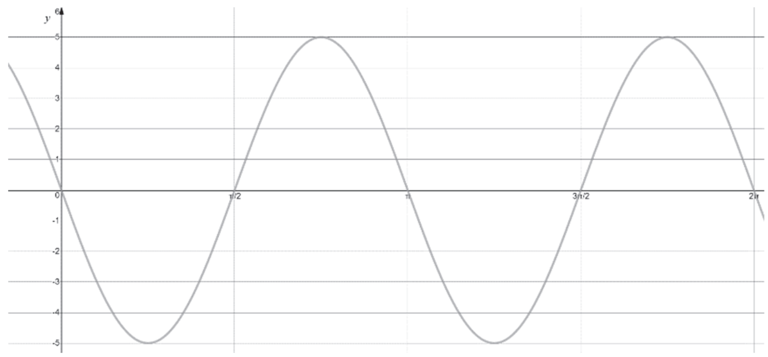


Section 6.6: Phase Shift; Sinusoidal Curve Fitting

Exploration 1. 5; π; yes; no; $\dfrac{\pi}{2}$;

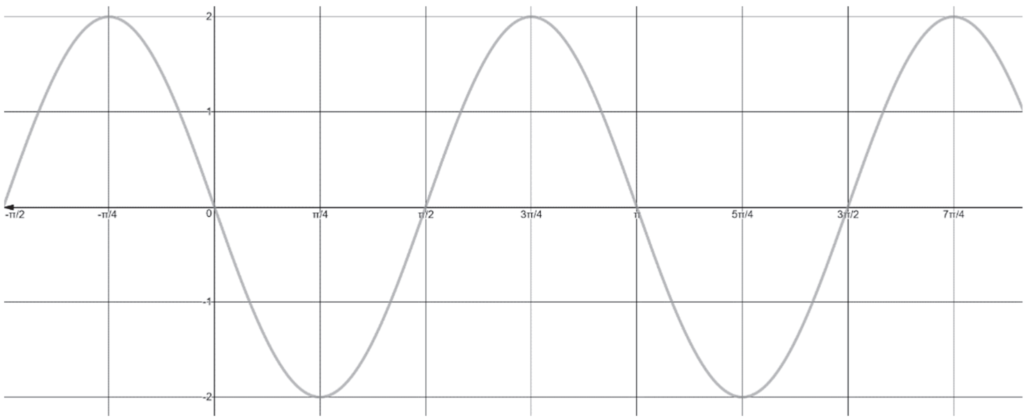
$\dfrac{\pi}{2}$

p. 182. $|A|; T; \dfrac{2\pi}{\omega}; \dfrac{\phi}{\omega}$



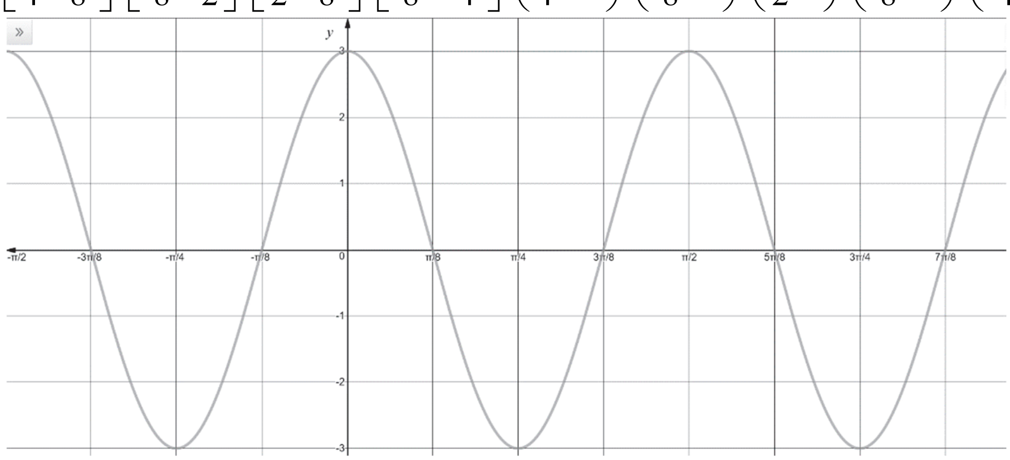
Example 1. 2; π; $-\dfrac{\pi}{2}; -\dfrac{\pi}{2}; -\dfrac{\pi}{2}+\pi=\dfrac{\pi}{2}$;

$\left[-\dfrac{\pi}{2},-\dfrac{\pi}{4}\right],\left[-\dfrac{\pi}{4},0\right],\left[0,\dfrac{\pi}{4}\right],\left[\dfrac{\pi}{4},\dfrac{\pi}{2}\right];\left(-\dfrac{\pi}{2},0\right),\left(-\dfrac{\pi}{4},2\right),(0,0),\left(\dfrac{\pi}{4},-2\right),\left(\dfrac{\pi}{2},0\right)$



Example 2. $3; \dfrac{\pi}{2}; \dfrac{\pi}{4}; \dfrac{\pi}{4}; \dfrac{\pi}{4}+\dfrac{\pi}{2}=\dfrac{3\pi}{4}$;

$\left[\dfrac{\pi}{4},\dfrac{3\pi}{8}\right],\left[\dfrac{3\pi}{8},\dfrac{\pi}{2}\right],\left[\dfrac{\pi}{2},\dfrac{5\pi}{8}\right],\left[\dfrac{5\pi}{8},\dfrac{3\pi}{4}\right];\left(\dfrac{\pi}{4},-3\right),\left(\dfrac{3\pi}{8},0\right),\left(\dfrac{\pi}{2},3\right),\left(\dfrac{5\pi}{8},0\right),\left(\dfrac{3\pi}{4},-3\right)$



p. 183. The graph would shift up 2 units;

Steps for Graphing Sinusoidal Functions $y = A\sin(\omega x - \phi) + B$ or $y = A\cos(\omega x - \phi) + B$

STEP 1: Determine the amplitude $|A|$, period $T = \frac{2\pi}{\omega}$, and phase shift $\frac{\phi}{\omega}$.

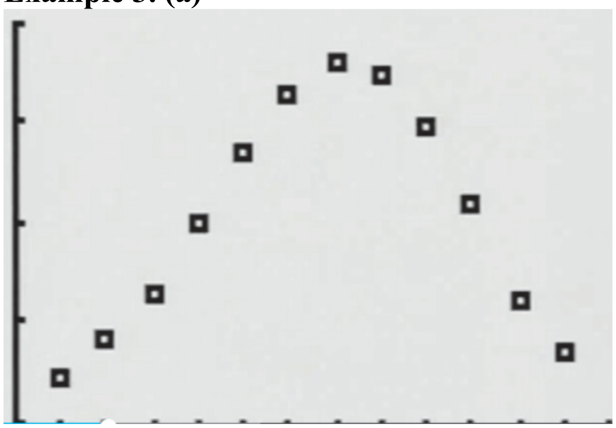
STEP 2: Determine the starting point of one cycle of the graph, $\frac{\phi}{\omega}$. Determine the ending point of one cycle of the graph, $\frac{\phi}{\omega} + \frac{2\pi}{\omega}$. Divide the interval $\left[\frac{\phi}{\omega}, \frac{\phi}{\omega} + \frac{2\pi}{\omega}\right]$ into four subintervals, each of length $\frac{2\pi}{\omega} \div 4$.

STEP 3: Use the endpoints of the subintervals to find the five key points on the graph.

STEP 4: Plot the five key points, and connect them with a sinusoidal graph to obtain one cycle of the graph. Extend the graph in each direction to make it complete.

STEP 5: If $B \neq 0$, apply a vertical shift.

Example 3. (a)



(b) $\frac{56.0-24.24}{2}$;15.9 **(c)** $\frac{56.0+24.24}{2}$;40.12 **(d)** 12; $\frac{\pi}{6}$ **(e)** 3; 7; 4 units

(f) $y = 15.9\sin\left(\frac{\pi}{6}(x-4)\right) + 40.12$

CHAPTER 7: Analytic Trigonometry

Section 7.1: The Inverse Sine, Cosine, and Tangent Function

p. 185. x; $f(f^{-1}(x))$; range of f^{-1}; domain of f^{-1}; symmetric; $y = x$; $x = f(y)$; $y = f^{-1}(x)$

Exploration 1. Fails the horizontal line test; Restrict the domain; $x = \sin y$; $y = f^{-1}(x) = \sin^{-1}(x)$

Example 1. (a) $\frac{\sqrt{3}}{2}$ **(b)** -$\frac{\pi}{3}$

Example 2. (a) I **(b)** radian; 0.64 **(c)** IV **(d)** radian; -0.73

Example 3. (a) $\frac{\pi}{12}$ **(b)** -0.4 **(c)** $\frac{2\pi}{5}$ **(d)** Use reference angle of $\frac{3\pi}{5}$ to draw diagram

Exploration 2. Fails the horizontal line test; Restrict the domain; $x = \cos y$; $y = f^{-1}(x) = \cos^{-1}(x)$

Example 4. (a) $-\frac{\sqrt{2}}{2}$ **(b)** $\frac{3\pi}{4}$

Example 5. (a) I **(b)** radian; 0.93 **(c)** radian; undefined because the domain of the cosine inverse function is [-1,1]

Example 6. (a) $\frac{2\pi}{3}$ **(b)** $\frac{\pi}{5}$ **(c)** 0.6 **(d)** undefined; 1.8 is not in the domain

Exploration 3. Fails the horizontal line test; Restrict the domain; $x = \tan y$; $y = f^{-1}(x) = \tan^{-1}(x)$

Example 7. (a) $-\frac{\sqrt{3}}{3}$ **(b)** $-\frac{\pi}{6}$

Example 8. (a) $f^{-1} = -\frac{x-2}{3}$ **(b)** range of f is [-1,5]; range of f^{-1} is [0, π]

Example 9. (a) $\tan^{-1} x = -\frac{\pi}{4}$ **(b)** {-1}

Section 7.2: The Inverse Trigonometric Functions (Continued)

p. 192. 1; 0, π; 1; $-\frac{\pi}{2};\frac{\pi}{2};-\infty;\infty;0;\pi$

Example 1. $\frac{\sqrt{3}}{3}$; I; $\frac{\pi}{3}$

Exploration 1. [0, π] except where undefined; cosine

$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ except where undefined; sine

[0, π] except where undefined; cosine

Example 2. (a) 1.37 **(b)** undefined **(c)** 2.16

Exploration 2. $\frac{3}{4}$; first; the domain restriction limits the angle from $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ and since the tangent is positive, the angle must be in quadrant I; $x = 4; y = 3; r = 5; \frac{4}{5}$

Example 3. $\frac{-2\sqrt{21}}{21}$

Exploration 3. $u;-\frac{\pi}{2};\frac{\pi}{2};-\infty,\infty$; $1+\tan^2\theta$; $\sqrt{1+\tan^2\theta};\sqrt{1+\tan^2\theta};\sqrt{1+\tan^2\theta}$

Section 7.3: Trigonometric Equations

Example 1. (a) not a solution **(b)** solution **(c)** periodic; infinite number of; $-\frac{7\pi}{6},\frac{5\pi}{6},\frac{13\pi}{6}$

Example 2. (a) $\cos\theta = \frac{\sqrt{3}}{2};\left\{\frac{\pi}{6},\frac{11\pi}{6}\right\}$

(b) $\theta = \frac{\pi}{6}+2\pi k, k$ is an integer; $\theta = \frac{11\pi}{6}+2\pi k, k$ is an integer

(c) $\left\{\frac{\pi}{6},\frac{11\pi}{6},\frac{13\pi}{6},\frac{23\pi}{6},-\frac{11\pi}{6},-\frac{\pi}{6},\frac{25\pi}{6},\frac{35\pi}{6}\right\}$

Exploration 1. When $\theta = \frac{3\pi}{2}$; $\frac{3\pi}{2}+2\pi k;\frac{\pi}{2}+\frac{2}{3}\pi k;\left\{\frac{\pi}{2},\frac{7\pi}{6},\frac{11\pi}{6}\right\}$

Exploration 2. Quadrant I or II; 0.41; yes, it is in Quadrant I; 0.41 or 2.73

Exploration 3. $2u^2-u-1=0;(2u+1)(u-1)=0;u=-\frac{1}{2};u=1;-\frac{1}{2};1;\left\{0,\frac{2\pi}{3},\frac{4\pi}{3}\right\}$

Example 3. (a) $1-\sin^2\theta-\sin^2\theta+\sin\theta=0$ **(b)** $\left\{\frac{\pi}{2},\frac{7\pi}{6},\frac{11\pi}{6}\right\}$

Exploration 4. (4.534, 4); 4.534

Section 7.4: Trigonometric Identities

Exploration 1. The horizontal line $y = 1$

p. 198. $f(x)=g(x)$; identity; conditional equation

Quotient Identities

$$\tan\theta=\frac{\sin\theta}{\cos\theta}\qquad \cot\theta=\frac{\cos\theta}{\sin\theta}$$

Reciprocal Identities

$$\csc\theta=\frac{1}{\sin\theta}\qquad \sec\theta=\frac{1}{\cos\theta}\qquad \cot\theta=\frac{1}{\tan\theta}$$

Pythagorean Identities

$$\sin^2\theta+\cos^2\theta=1\qquad \tan^2\theta+1=\sec^2\theta$$
$$\cot^2\theta+1=\csc^2\theta$$

Even–Odd Identities

$$\sin(-\theta)=-\sin\theta\qquad \cos(-\theta)=\cos\theta\qquad \tan(-\theta)=-\tan\theta$$
$$\csc(-\theta)=-\csc\theta\qquad \sec(-\theta)=\sec\theta\qquad \cot(-\theta)=-\cot\theta$$

Example 1.

$$\begin{aligned}\frac{\sin\theta}{1+\cos\theta}\cdot\frac{1-\cos\theta}{1-\cos\theta}&=\frac{\sin\theta(1-\cos\theta)}{(1+\cos\theta)(1+\cos\theta)}\\&=\frac{\sin\theta(1-\cos\theta)}{1-\cos\theta+\cos\theta-\cos^2\theta}\\&=\frac{\sin\theta(1-\cos\theta)}{1-\cos^2\theta}\\&=\frac{\sin\theta(1-\cos\theta)}{\sin^2\theta}\\&=\frac{1-\cos\theta}{\sin\theta}\end{aligned}$$

Example 2.

$$\begin{aligned}\frac{1}{1-\sin\theta}+\frac{1}{1+\sin\theta}&=\frac{1}{1-\sin\theta}\cdot\frac{1+\sin\theta}{1+\sin\theta}+\frac{1}{1+\sin\theta}\cdot\frac{1-\sin\theta}{1-\sin\theta}\\&=\frac{1+\sin\theta}{1-\sin^2\theta}+\frac{1-\sin\theta}{1-\sin^2\theta}\\&=\frac{2}{\cos^2\theta}\\&=2\sec^2\theta\end{aligned}$$

Example 3.

$$\begin{aligned}\frac{1-\cos^2 v}{\sin v+\cos v\sin v}&=\frac{(1+\cos v)(1-\cos v)}{\sin v(1+\cos v)}\\&=\frac{1-\cos v}{\sin v}\\&=\csc v-\cot v\end{aligned}$$

Example 4.

$$\sin\theta(\cot\theta+\tan\theta)=\sec\theta$$
$$\sin\theta\cot\theta+\sin\theta\tan\theta=\sec\theta$$
$$\sin\theta\cdot\frac{\cos\theta}{\sin\theta}+\sin\theta\cdot\frac{\sin\theta}{\cos\theta}=\sec\theta$$
$$\cos\theta+\frac{\sin^2\theta}{\cos\theta}=\sec\theta$$
$$\cos\theta\cdot\frac{\cos\theta}{\cos\theta}+\frac{\sin^2\theta}{\cos\theta}=\sec\theta$$
$$\frac{\cos^2\theta}{\cos\theta}+\frac{\sin^2\theta}{\cos\theta}=\sec\theta$$
$$\frac{1}{\cos\theta}=\sec\theta$$
$$\sec\theta=\sec\theta$$

Example 5.

$$\csc\theta-\cot\theta=\frac{\sin\theta}{1+\cos\theta}$$
$$\frac{1}{\sin\theta}-\frac{\cos\theta}{\sin\theta}=$$
$$\frac{1-\cos\theta}{\sin\theta}=$$
$$\frac{1-\cos\theta}{\sin\theta}\cdot\frac{1+\cos\theta}{1+\cos\theta}=$$
$$\frac{1-\cos^2\theta}{\sin\theta(1+\cos\theta)}=$$
$$\frac{\sin^2\theta}{\sin\theta(1+\cos\theta)}=$$
$$\frac{\sin\theta}{1+\cos\theta}=\frac{\sin\theta}{1+\cos\theta}$$

Example 6.

$$\frac{\sin^2\theta-\tan\theta}{\cos^2\theta-\cot\theta}=\tan^2\theta$$
$$\frac{\sin^2\theta-\frac{\sin\theta}{\cos\theta}}{\cos^2\theta-\frac{\cos\theta}{\sin\theta}}=$$
$$\frac{\sin^2\theta-\frac{\sin\theta}{\cos\theta}}{\cos^2\theta-\frac{\cos\theta}{\sin\theta}}\cdot\frac{\cos\theta}{\cos\theta}=$$
$$\frac{\cos\theta\sin^2\theta-\sin\theta}{\cos^3\theta-\frac{\cos^2\theta}{\sin\theta}}\cdot\frac{\sin\theta}{\sin\theta}=$$
$$\frac{\cos\theta\sin^3\theta-\sin^2\theta}{\sin\theta\cos^3\theta-\cos^2\theta}=$$
$$\frac{\sin^2\theta(\cos\theta\sin\theta-1)}{\cos^2\theta(\sin\theta\cos\theta-1)}=$$
$$\frac{\sin^2\theta}{\cos^2\theta}=$$
$$\tan^2\theta=\tan^2\theta$$

Section 7.5: Sum and Difference Formulas

p. 202. 45° - 30°

Exploration 1. $(\cos\alpha-\cos\beta)^2+(\sin\alpha-\sin\beta)^2$;

$\cos^2\alpha-2\cos\alpha\cos\beta+\cos^2\beta+\sin^2\alpha-2\sin\alpha\sin\beta+\sin^2\beta$;

$2-2\cos\alpha\cos\beta-2\sin\alpha\sin\beta$;

$\left(\cos(\alpha-\beta)-1\right)^2+\left(\sin(\alpha-\beta)-0\right)^2$;

$\cos^2(\alpha-\beta)-2\cos(\alpha-\beta)+1+\sin^2(\alpha-\beta)$;

$2-2\cos(\alpha-\beta)$;

$$\begin{aligned}\sqrt{2-2\cos(\alpha-\beta)}&=\sqrt{2-2\cos\alpha\cos\beta-2\sin\alpha\sin\beta}\\ 2-2\cos(\alpha-\beta)&=2-2\cos\alpha\cos\beta-2\sin\alpha\sin\beta\\ -2\cos(\alpha-\beta)&=-2\cos\alpha\cos\beta-2\sin\alpha\sin\beta\\ \cos(\alpha-\beta)&=\cos\alpha\cos\beta+\sin\alpha\sin\beta\end{aligned}$$

$$\begin{aligned}\cos(\alpha+\beta)&=\cos(\alpha-(-\beta))\\ &=\cos\alpha\cos(-\beta)+\sin\alpha\sin(-\beta)\\ &=\cos\alpha\cos\beta-\sin\alpha\sin\beta\end{aligned}$$

Example 1. $\dfrac{\sqrt{6}+\sqrt{2}}{4}$

Example 2. $\cos(40°+80°);120°;-\dfrac{1}{2}$

Example 3. $\sin\theta$

Exploration 2. $\alpha;\alpha;\cos\theta$

Exploration 3. $\theta;\beta;\beta;\sin\alpha\cos\beta+\cos\alpha\sin\beta$

Exploration 4. $\sin\alpha\cos(-\beta)+\cos\alpha\sin(-\beta);\sin\alpha\cos\beta-\cos\alpha\sin\beta$

Example 4. $-\left(\dfrac{\sqrt{6}+\sqrt{2}}{4}\right)$

Example 5. (a) $-\dfrac{4}{5}$ **(b)** $\dfrac{2\sqrt{5}}{5}$ **(c)** $-\dfrac{\sqrt{5}}{5}$ **(d)** $\dfrac{2\sqrt{5}}{5}$

Example 6. $\dfrac{\tan 2\pi-\tan\theta}{1+\tan 2\pi\cdot\tan\theta}$

Example 7. (a) $\sin^{-1}\dfrac{2}{3};\tan^{-1}\left(-\dfrac{3}{4}\right)$ **(b)** $\sin\alpha=\dfrac{2}{3};\cos\alpha=\dfrac{\sqrt{5}}{3};\sin\beta=-\dfrac{3}{5};\cos\beta=\dfrac{4}{5}$

(c) $\dfrac{4\sqrt{15}+6}{15}$

Example 8. **(b)** $-\frac{1}{\sqrt{2}}=-\frac{\sqrt{2}}{2}=\sin\theta\cos\phi+\cos\theta\sin\phi=\sin(\theta+\phi)$ **(c)** $\left\{0,\frac{\pi}{2}\right\}$

Section 7.6: Double-angle and Half-angle Formulas

Exploration 1. **(a)** θ; θ; $\sin\theta\cos\theta+\cos\theta\sin\theta; 2\sin\theta\cos\theta$

(b) $\theta; \theta \cos\theta\cos\theta-\sin\theta\sin\theta; \cos^2\theta-\sin^2\theta$ **(c)** $\cos^2\theta-\sin^2\theta; (1-\sin^2\theta)-\sin^2\theta; 1-2\sin^2\theta$

(d) $\cos^2\theta-\sin^2\theta=\cos^2\theta-(1-\cos^2\theta); 2\cos^2\theta-1$

Example 1. **(a)** $\frac{4\sqrt{21}}{25}$ **(b)** $-\frac{17}{25}$

Exploration 2. **(a)** θ; θ; $\frac{\tan\theta+\tan\theta}{1-\tan\theta\tan\theta};\frac{2\tan\theta}{1-\tan^2\theta}$

(b)

$$\begin{aligned}
\sin(3\theta) &= \sin(2\theta+\theta)\\
&=\sin(2\theta)\cdot\cos\theta+\cos(2\theta)\cdot\sin\theta\\
&=2\sin\theta\cos\theta\cdot\cos\theta+\left(2\cos^2\theta-1\right)\cdot\sin\theta\\
&=2\cos^2\theta\cdot\sin\theta+2\cos^2\theta\sin\theta-\sin\theta\\
&=4\cos^2\theta\sin\theta-\sin\theta\\
&=4\sin\theta\left(1-\sin^2\theta\right)-\sin\theta\\
&=4\sin\theta-4\sin^3\theta-\sin\theta\\
&=3\sin\theta-4\sin^3\theta
\end{aligned}$$

Exploration 3. **(a)** $2\sin^2\theta=1-\cos(2\theta); \sin^2\theta=\frac{1-\cos(2\theta)}{2}$

(b) $\cos(2\theta)+1=2\cos^2\theta; \frac{\cos(2\theta)+1}{2}=\cos^2\theta$

(c) $\frac{\sin\theta}{\cos\theta}\cdot\frac{\sin\theta}{\cos\theta};\frac{\frac{1-\cos(2\theta)}{2}}{\frac{\cos(2\theta)+1}{2}};\frac{1-\cos(2\theta)}{\cos(2\theta)+1}$

Exploration 4. (a) $\dfrac{1-\cos\left(2\cdot\dfrac{\alpha}{2}\right)}{2};\dfrac{1-\cos\alpha}{2}$ **(b)** $\dfrac{\cos\left(2\cdot\dfrac{\alpha}{2}\right)+1}{2};\dfrac{1+\cos\alpha}{2}$

(c) $\dfrac{1-\cos\left(2\cdot\dfrac{\alpha}{2}\right)}{\cos\left(2\cdot\dfrac{\alpha}{2}\right)+1};\dfrac{1-\cos\alpha}{1+\cos\alpha}$

p. 210.

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$$
$$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$$
$$\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$$

Example 2. (a) $\pm\dfrac{\sqrt{2-\sqrt{2}}}{2}$ **(b)** first; positive; positive

Example 3. (a) $\pm\dfrac{\sqrt{2-\sqrt{3}}}{2}$ **(b)** first; positive; positive

Example 4. (a) second **(b)** first; positive; since $\dfrac{\pi}{2}<\alpha<\pi, \dfrac{\pi}{4}<\dfrac{\alpha}{2}<\dfrac{\pi}{2}$ **(c)** $-\dfrac{5\sqrt{26}}{26}$

(d) $\sqrt{\dfrac{26+130\sqrt{26}}{52}}$

p. 211.

$$\tan\frac{\alpha}{2} = \frac{1-\cos\alpha}{\sin\alpha} = \frac{\sin\alpha}{1+\cos\alpha}$$

Section 7.7: Product-to-Sum and Sum-to-Product Formulas

p. 212.

$$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$$
$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$$
$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$$

Example 1. (a) $\frac{1}{2}\left[\cos(4\theta)-\cos(10\theta)\right]$ **(b)** $\frac{1}{2}\left[\cos(4\theta)+\cos(6\theta)\right]$ **(c)** $\frac{1}{2}\left[\sin(3\theta)-\sin(2\theta)\right]$

p. 213.

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$
$$\sin\alpha - \sin\beta = 2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}$$
$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$
$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

Example 2. (a) $-2\sin(\theta)\cos(5\theta)$ **(b)** $2\cos(5\theta)\cos(3\theta)$

CHAPTER 8: Applications of Trigonometric Functions

Section 8.1: Right Triangle Trigonometry; Applications

Exploration 1. 90; hypotenuse; legs; $a^2 + b^2 = c^2$; radius;

$\frac{b}{c};\frac{a}{c};\frac{b}{a}$

$\frac{c}{b};\frac{c}{a};\frac{a}{b}$

a = adjacent angle θ; b = opposite angle θ

p. 214.

$$\sin\theta = \frac{opposite}{hypotenuse} = \frac{b}{c} \qquad \cos\theta = \frac{adjacent}{hypotenuse} = \frac{a}{c} \qquad \tan\theta = \frac{opposite}{adjacent} = \frac{b}{a}$$

$$\csc\theta = \frac{hypotenuse}{opposite} = \frac{c}{b} \qquad \sec\theta = \frac{hypotenuse}{adjacent} = \frac{c}{a} \qquad \cot\theta = \frac{adjacent}{opposite} = \frac{a}{b}$$

SOH-CAH-TOA

Example 1. adjacent; $\sqrt{51}$;

$$\sin\theta = \frac{7}{10};\cos\theta = \frac{\sqrt{51}}{10};\tan\theta = \frac{7\sqrt{51}}{51};\csc\theta = \frac{10}{7};\sec\theta = \frac{10\sqrt{51}}{51};\cot\theta = \frac{\sqrt{51}}{7}$$

Exploration 2. $\frac{b}{c};\frac{a}{c};\frac{a}{c};\frac{b}{c};\sin B = \cos A;\cos B = \sin A$; cofunctions; cotangent; secant; complimentary; 90°; Cofunctions; complimentary

Example 2. (a) cos55° **(b)** $\cot\frac{\pi}{6}$ **(c)** $\csc\frac{5\pi}{12}$

Example 3. smallest; largest

Example 4. $c = 8.06$; $A = 29.7°$; $B = 60.3°$

Example 5. 130 meters

Example 6. (a) ≈ 103 ft **(b)** Adds 5 feet to the height of the tree **(c)** Pythagorean Theorem or Trigonometric Ratios

Example 7. ≈ 1939 feet

Example 8. N43.4°W

Section 8.2: The Law of Sines

Exploration 1. Oblique; acute; acute; obtuse

p. 219.

Case 1: One side and two angles are known (ASA or SAA).
Case 2: Two sides and the angle opposite one of them are known (SSA).
Case 3: Two sides and the included angle are known (SAS).
Case 4: Three sides are known (SSS).

Exploration 2. $\frac{h}{c}; \frac{h}{a}; h = c\sin A; h = a\sin C; c\sin A = a\sin C; \frac{\sin A}{a} = \frac{\sin C}{c}$

Exploration 3.

$$\begin{aligned}\sin(180° - A) &= \sin 180° \cos A - \cos 180° \sin A \\ &= 0 - (-1)\sin A \\ &= \sin A\end{aligned}$$

$\sin A = \frac{h}{c}$

Example 1. SAA; **.** C = 80°; b = 11.52; c = 11.52

Example 2. ASA; **.** A = 70°; b = 2.70; c = 6.36

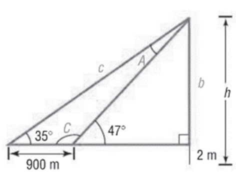
p. 221. Ambiguous; two; one; none

Example 3. A = 20.1°; C = 124.9°; c = 7.15

Example 4. No triangle

Example 5. A = 25.6° or 154.4°; C = 139.4° or 10.6°; b = 15.07 or 4.25

Example 6.



≈1818 meters

Example 7. 1490.48 feet

Example 8. ≈37 feet

Example 9. (a) Sketches will vary **(b)** Since $A > B$, $a > b$ **(c)** ≈198.90 feet closer

Section 8.3: The Law of Cosines

Exploration 1. Same image as p. 287

p. 224.

$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$

Exploration 2. $(a\cos C, a\sin C);(b,0)$

$$c = \sqrt{(b - a\cos C)^2 + (0 - a\sin C)^2}$$

$$\begin{aligned} c^2 &= (b - a\cos C)^2 + (0 - a\sin C)^2 \\ &= b^2 - 2ab\cos C + a^2\cos^2 C + a^2\sin^2 C \\ &= b^2 - 2ab\cos C + a^2\left(\cos^2 C + \sin^2 C\right) \\ &= a^2 + b^2 - 2ab\cos C \end{aligned}$$

Example 1. $a = 4.84$; $B = 36.3°$; $C = 108.7°$

Example 2. $C = 38.6°$; $B = 92.9°$; $A = 48.5°$

Example 3. (a) ≈96 miles **(b)** 33° **(c)** 0.4 hours or 24 minutes

Section 8.4: Area of a Triangle

Exploration 1. sine; $\sin C = \frac{h}{a}; h = a\sin C; a\sin C;$ Area $= \frac{1}{2}ab\sin C$

p. 226. Product; two; sine

Example 1. ≈12.86 square units

p. 227. $K = \sqrt{s(s-a)(s-b)(s-c)}; s = \frac{1}{2}(a+b+c)$

Example 2. ≈9.56 square units

Example 3. 212.7 square feet

Example 4. 5 cans of paint

Section 8.5: Simple Harmonic Motion; Damped Motion; Combining Waves

Exploration 1. equilibrium; height; time; amplitude; period

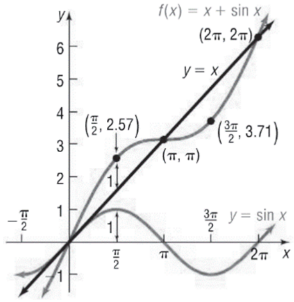
Directly; proportional; negative; $x = a\cos(\omega t); y = a\sin(\omega t)$

p. 229. $d = a\cos\omega t; d = a\sin\omega t;$ amplitude; period; period

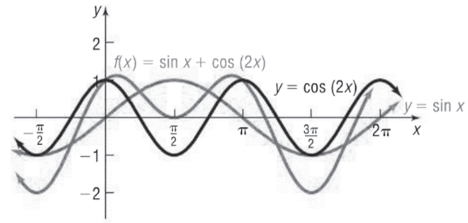
Example 1. $d = -5\cos\left(\frac{2\pi}{3}t\right)$

Example 2. **(a)** Simple Harmonic Motion **(b)** 5 **(c)** $\frac{2\pi}{3}$ **(d)** $\frac{3}{2\pi}$ osc./seconds

Example 3.



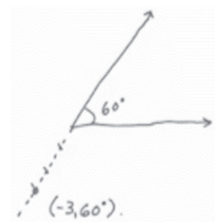
Example 4.



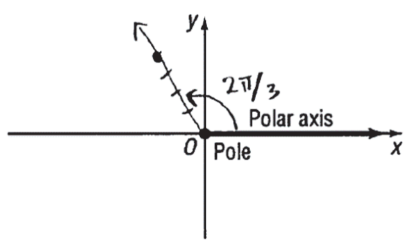
CHAPTER 9: Polar Coordinates; Vectors

Section 9.1: Polar Coordinates

Exploration 1. (r, θ); real number line; radians; if r is negative, draw the angle and go $|r|$ units in the direction opposite the terminal side of the angle;



Example 1. (a)



(b) $\left(-4, \frac{5\pi}{3}\right), \left(4, \frac{8\pi}{3}\right), \left(4, -\frac{4\pi}{3}\right)$

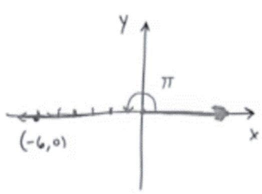
p. 232. $(r, \theta + 2\pi k), (-r, \theta + \pi + 2\pi k)$

Exploration2. $\sin\theta = \frac{y}{r}; \cos\theta = \frac{x}{r}; y = r\sin\theta; x = r\cos\theta$

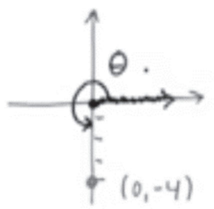
$y = r\sin\theta; x = r\cos\theta$

Example 2. (a) $\left(\frac{5\sqrt{2}}{2}, \frac{5\sqrt{2}}{2}\right)$ **(b)** $\left(\sqrt{3}, -1\right)$ **(c)** yes, because they are in the same quadrants

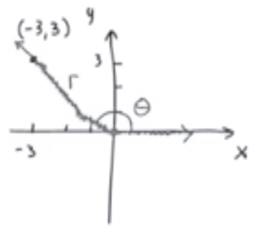
Exploration3. (6, π)



$\left(4,\frac{3\pi}{2}\right);\left(4,-\frac{\pi}{2}\right);\left(-4,\frac{\pi}{2}\right);\left(-4,-\frac{3\pi}{2}\right)$



Exploration4. II; $r^2 = x^2 + y^2; r = 3\sqrt{2}; \tan\theta = \frac{y}{x}; \pi; \theta = \frac{3\pi}{4}$



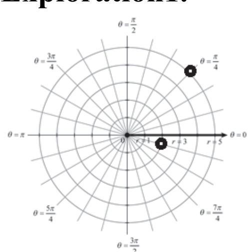
Exploration 5. square; $r; x^2 - x + y^2 = 0$

Example 3. (a) $x^2 + y^2 = 25$ **(b)** $x^2 + (y-2)^2 = 4$

Example 4. (a) $r\sin\theta = 3$ **(b)** $(r\sin\theta)^2 = 3r\cos\theta$

Section 9.2: Polar Equations and Graphs

Exploration1.



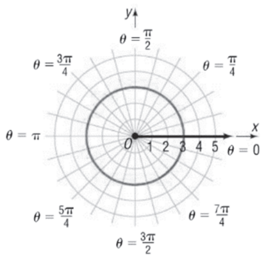
(3.54, 3.54); (1.73, -1); $\left(4.24,\frac{3\pi}{4}\right)$; the answers are verified because both the polar and rectangular points lie on the same point.

p. 236. polar coordinates; satisfy

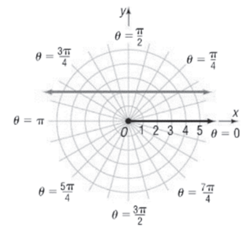
Example 1. (a) $x^2 + y^2 = 9$ **(b)** circle

Example 2. (a) $y = 2$; **(b)** horizontal line

(c)

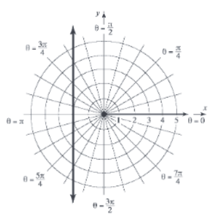


(c)



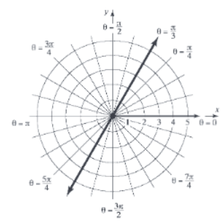
Example 3. (a) $x = -2$; **(b)** vertical line

(c)

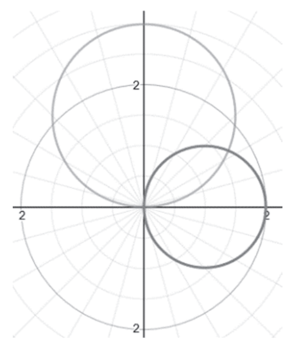


Example 4. (a) $y=\sqrt{3}x$ **(b)** line

(c)



Example 5. (a) $x^2+\left(y-\frac{3}{2}\right)^2=\frac{9}{4};(x-1)^2+y^2=1$ **(b)** circle **(c)**



p. 238. $r;\theta$; polar; polar; $\theta;\theta;(\theta\text{step})$; square; θ

Example 6. Technique will vary depending on graphing utility.

Exploration2. x-axis; y-axis; origin

p. 239. $\theta;-\theta;\theta;-\theta;r;-r;\theta;\theta+\pi$

An equation may fail these tests and still have a graph that is symmetric with respect to the polar axis, the line $\theta=\frac{\pi}{2}$, or the pole.

Example 7. (a) $a \pm a\cos\theta, a>0$

(b)

Polar axis: Replace θ by $-\theta$. The result is $r = 2 + 2\cos(-\theta) = 2 + 2\cos\theta$.

The graph is symmetric with respect to the polar axis.

The line $\theta = \frac{\pi}{2}$: Replace θ by $\pi - \theta$.

$$\begin{aligned} r &= 2 + 2\cos(\pi - \theta) \\ &= 2 + 2[\cos(\pi)\cos\theta + \sin(\pi)\sin\theta] \\ &= 2 + 2(-\cos\theta + 0) \\ &= 2 - 2\cos\theta \end{aligned}$$

The test fails.

The pole: Replace r by $-r$.

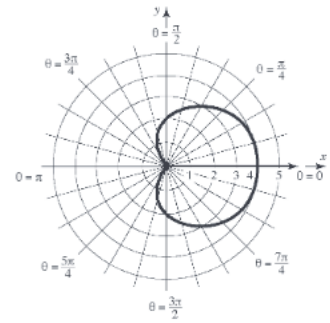
$-r = 2 + 2\cos\theta$. The test fails.

Due to symmetry with respect to the polar axis, assign values to θ from 0 to π.

(c)

θ	$r = 2 + 2\cos\theta$
0	4
$\frac{\pi}{6}$	$2 + \sqrt{3} \approx 3.7$
$\frac{\pi}{3}$	3
$\frac{\pi}{2}$	2
$\frac{2\pi}{3}$	1
$\frac{5\pi}{6}$	$2 - \sqrt{3} \approx 0.3$
π	0

(d)



Example 8.

(a)

Polar axis: Replace θ by $-\theta$. The result is $r = 4 - 2\cos(-\theta) = 4 - 2\cos\theta$. The graph is symmetric with respect to the polar axis.

The line $\theta = \frac{\pi}{2}$: Replace θ by $\pi - \theta$.

$$\begin{aligned} r &= 4 - 2\cos(\pi - \theta) \\ &= 4 - 2[\cos(\pi)\cos\theta + \sin(\pi)\sin\theta] \\ &= 4 - 2(-\cos\theta + 0) \\ &= 4 + 2\cos\theta \end{aligned}$$

The test fails.

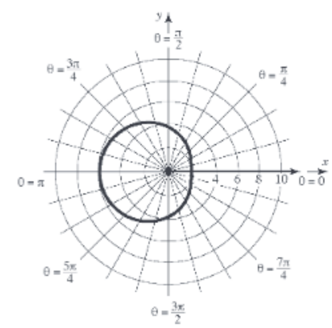
The pole: Replace r by $-r$. $-r = 4 - 2\cos\theta$. The test fails.

Due to symmetry with respect to the polar axis, assign values to θ from 0 to π.

(b)

θ	$r = 4 - 2\cos\theta$
0	2
$\frac{\pi}{6}$	$4 - \sqrt{3} \approx 2.3$
$\frac{\pi}{3}$	3
$\frac{\pi}{2}$	4
$\frac{2\pi}{3}$	5
$\frac{5\pi}{6}$	$4 + \sqrt{3} \approx 5.7$
π	6

(c)



Example 9.

(a)

Polar axis: Replace θ by $-\theta$. The result is $r = 1 + 2\sin(-\theta) = 1 - 2\sin\theta$. The test fails.

The line $\theta = \frac{\pi}{2}$: Replace θ by $\pi - \theta$.

$$\begin{aligned} r &= 1 + 2\sin(\pi - \theta) \\ &= 1 + 2[\sin(\pi)\cos\theta - \cos(\pi)\sin\theta] \\ &= 1 + 2(0 + \sin\theta) \\ &= 1 + 2\sin\theta \end{aligned}$$

The graph is symmetric with respect to the line $\theta = \frac{\pi}{2}$.

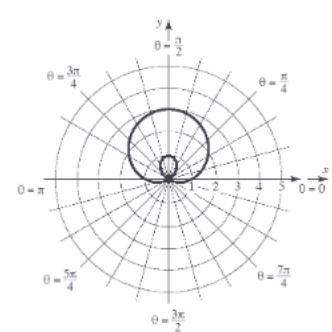
The pole: Replace r by $-r$. $-r = 1 + 2\sin\theta$. The test fails.

Due to symmetry with respect to the line $\theta = \frac{\pi}{2}$, assign values to θ from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$.

(b)

θ	$r = 1 + 2\sin\theta$
$-\frac{\pi}{2}$	-1
$-\frac{\pi}{3}$	$1 - \sqrt{3} \approx -0.7$
$-\frac{\pi}{6}$	0
0	1
$\frac{\pi}{6}$	2
$\frac{\pi}{3}$	$1 + \sqrt{3} \approx 2.7$
$\frac{\pi}{2}$	3

(c)



Example 10.

(a)

Polar axis: Replace θ by $-\theta$.

$r = 3\cos(2(-\theta)) = 3\cos(-2\theta) = 3\cos(2\theta)$. The graph is symmetric with respect to the polar axis.

The line $\theta = \frac{\pi}{2}$: Replace θ by $\pi - \theta$.

$$\begin{aligned} r &= 3\cos[2(\pi-\theta)] \\ &= 3\cos(2\pi - 2\theta) \\ &= 3[\cos(2\pi)\cos(2\theta) + \sin(2\pi)\sin(2\theta)] \\ &= 3(\cos 2\theta + 0) \\ &= 3\cos(2\theta) \end{aligned}$$

The graph is symmetric with respect to the line $\theta = \frac{\pi}{2}$.

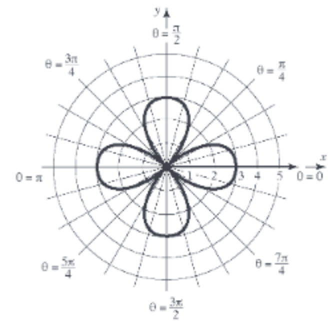
The pole: Since the graph is symmetric with respect to both the polar axis and the line $\theta = \frac{\pi}{2}$, it is also symmetric with respect to the pole.

Due to symmetry, assign values to θ from 0 to $\frac{\pi}{2}$.

(b)

θ	$r = 3\cos(2\theta)$
	3
$\frac{\pi}{6}$	$\frac{3}{2}$
$\frac{\pi}{4}$	0
$\frac{\pi}{3}$	$-\frac{3}{2}$
$\frac{\pi}{2}$	-3

(c)



Example 11.

(a)

Polar axis: Replace θ by $-\theta$.

$r^2 = 9\cos(2(-\theta)) = 9\cos(-2\theta) = 9\cos(2\theta)$. The graph is symmetric with respect to the polar axis.

The line $\theta = \frac{\pi}{2}$: Replace θ by $\pi - \theta$.

$$\begin{aligned} r^2 &= 9\cos[2(\pi-\theta)] \\ &= 9\cos(2\pi - 2\theta) \\ &= 9[\cos(2\pi)\cos 2\theta + \sin(2\pi)\sin 2\theta] \\ &= 9(\cos 2\theta + 0) \\ &= 9\cos(2\theta) \end{aligned}$$

The graph is symmetric with respect to the line $\theta = \frac{\pi}{2}$.

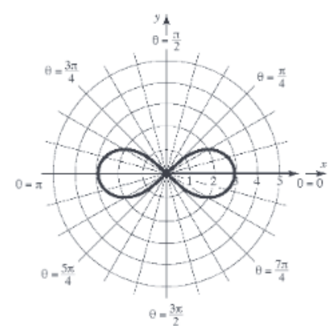
The pole: Since the graph is symmetric with respect to both the polar axis and the line $\theta = \frac{\pi}{2}$, it is also symmetric with respect to the pole.

Due to symmetry, assign values to θ from 0 to $\frac{\pi}{2}$.

(b)

θ	$r = \pm\sqrt{9\cos(2\theta)}$
0	± 3
$\frac{\pi}{6}$	$\pm\frac{3\sqrt{2}}{2} \approx \pm 2.1$
$\frac{\pi}{4}$	0
$\frac{\pi}{3}$	undefined
$\frac{\pi}{2}$	undefined

(c)



Section 9.3: The Complex Plane; De Moivre's Theorem

Exploration 1. real; imaginary; 3; -2; magnitude; modulus; $|z|$; $\theta \approx -0.588$; $|z| = \sqrt{13}$

p. 246. $|z|$; $\sqrt{x^2 + y^2}$; $\sqrt{z \cdot \bar{z}}$

Exploration 2. $r\cos\theta; r\sin\theta; (r\cos\theta) + (r\sin\theta)i = r(\cos\theta + i\sin\theta)$

Example 1. (a) Quadrant IV **(b)** $r=6; \theta=\frac{5\pi}{3}; z=6\left(\cos\frac{5\pi}{3}+i\sin\frac{5\pi}{3}\right)$

p. 246. $e^{i\theta}; re^{i\theta}; z=6e^{i\frac{5\pi}{3}}$

Example 2. (a) $r=3; \theta=60°$ **(b)** Quadrant I **(c)** $z=\frac{3}{2}+\frac{3\sqrt{3}}{2}i$

p. 247. $z_1z_2=r_1e^{i\theta_1}\cdot r_2e^{i\theta_2}=r_1r_2e^{i(\theta_1+\theta_2)}; \frac{z_1}{z_2}=\frac{r_1e^{i\theta_1}}{r_2e^{i\theta_2}}=\frac{r_1}{r_2}e^{i(\theta_1-\theta_2)}$

Example 3. (a) $16e^{i60°}=16[\cos 60°+i\sin 60°]$ **(b)** $4e^{i20°}=4[\cos 20°+i\sin 20°]$

Exploration 3. $r^2e^{i(2\theta)}; r^3e^{i(3\theta)}; r^4e^{i(4\theta)}$

p. 248. $r^n e^{i(n\theta)}$

Example 4. (a) Quadrant IV; $1-\sqrt{3}i=2\left[\cos\frac{5\pi}{3}+i\sin\frac{5\pi}{3}\right]=2e^{i\frac{5\pi}{3}}$

(b)

$$\left(1-\sqrt{3}i\right)^{10}=\left[2\left(\cos\frac{5\pi}{3}+i\sin\frac{5\pi}{3}\right)\right]^{10}=\left(2e^{i\frac{5\pi}{3}}\right)^{10}=2^{10}e^{i\left(10\cdot\frac{5\pi}{3}\right)}=2^{10}\left[\cos\left(10\cdot\frac{5\pi}{3}\right)+i\sin\left(10\cdot\frac{5\pi}{3}\right)\right]$$

(c) $-512+512\sqrt{3}i$

p. 249. Two; two; $z_k=\sqrt[n]{r}e^{i\frac{1}{n}(\theta+2k\pi)}$

Example 5. (a) Quadrant II; $-2+2i=\sqrt{8}e^{i\frac{3\pi}{4}}=\sqrt{8}\left[\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right]$

(b) $z_k=\sqrt[4]{\sqrt{8}}e^{i\frac{1}{4}\cdot\frac{3\pi+8k\pi}{4}}=\sqrt[4]{\sqrt{8}}\left[\cos\left(\frac{3\pi}{16}+\frac{2\pi k}{4}\right)+i\sin\left(\frac{3\pi}{16}+\frac{2\pi k}{4}\right)\right], k=0,1,2,3$

(c) $z_0=\sqrt[4]{\sqrt{8}}\left[\cos\left(\frac{3\pi}{16}\right)+i\sin\left(\frac{3\pi}{16}\right)\right]$; $z_1=\sqrt[4]{\sqrt{8}}\left[\cos\left(\frac{3\pi}{16}+\frac{\pi}{2}\right)+i\sin\left(\frac{3\pi}{16}+\frac{\pi}{2}\right)\right]$;

$z_2=\sqrt[4]{\sqrt{8}}\left[\cos\left(\frac{3\pi}{16}+\pi\right)+i\sin\left(\frac{3\pi}{16}+\pi\right)\right]$; $z_3=\sqrt[4]{\sqrt{8}}\left[\cos\left(\frac{3\pi}{16}+\frac{3\pi}{2}\right)+i\sin\left(\frac{3\pi}{16}+\frac{3\pi}{2}\right)\right]$

(d) $-2+2i$ **(e)** The angles are spaced equal distance from each other around the circle.

Section 9.4: Vectors

Exploration 1. Magnitude; direction; magnitude; direction; magnitude; direction; begin; end; $\vec{v}$; terminal; initial; $\vec{v}; \vec{w}$

Exploration 2. ≈3.85 miles; yes, forms a parallelogram; commutative

p. 251. commutative; $\vec{v}+\vec{w}=\vec{w}+\vec{v}$
associative; $\vec{u}+\left(\vec{v}+\vec{w}\right)=\left(\vec{u}+\vec{v}\right)+\vec{w}$

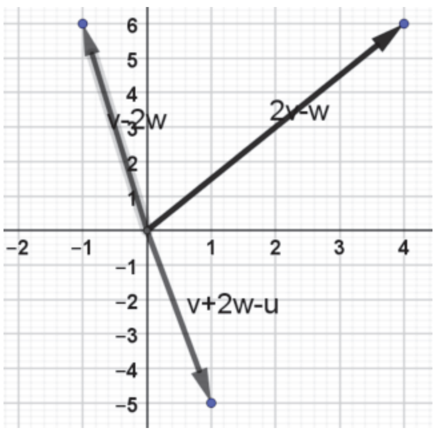
$\vec{v}+0=0+\vec{v}=\vec{v}$; magnitude; opposite; $\vec{v}+\left(-\vec{v}\right)=0; \vec{v}-\vec{w}=\vec{v}+\left(-\vec{w}\right)$

p. 252. real; scalar product; $\alpha>0$; same; $\alpha<0$; opposite; $\alpha=0$; $\alpha\vec{v}=0$

Example 1.

$$0\mathbf{v}=\mathbf{0} \qquad 1\mathbf{v}=\mathbf{v} \qquad -1\mathbf{v}=-\mathbf{v}$$
$$(\alpha+\beta)\mathbf{v}=\alpha\mathbf{v}+\beta\mathbf{v} \qquad \alpha(\mathbf{v}+\mathbf{w})=\alpha\mathbf{v}+\alpha\mathbf{w}$$
$$\alpha(\beta\mathbf{v})=(\alpha\beta)\mathbf{v}$$

Example 2.



p. 252

Properties of $\|\mathbf{v}\|$

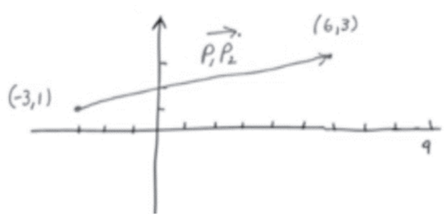
If $\mathbf{v}$ is a vector and if α is a scalar, then

(a) $\|\mathbf{v}\|\geq 0$ (b) $\|\mathbf{v}\|=0$ if and only if $\mathbf{v}=\mathbf{0}$

(c) $\|-\mathbf{v}\|=\|\mathbf{v}\|$ (d) $\|\alpha\mathbf{v}\|=|\alpha|\|\mathbf{v}\|$

; 1

p. 253. (0, 0)
Exploration 3. $\langle 9,2\rangle$; magnitude; direction; $\langle x_2-x_1, y_2-y_1\rangle$



p. 254.

Equality of Vectors

Two vectors $\mathbf{v}$ and $\mathbf{w}$ are equal if and only if their corresponding components are equal. That is,

If $\mathbf{v}=\langle a_1, b_1\rangle$ and $\mathbf{w}=\langle a_2, b_2\rangle$
then $\mathbf{v}=\mathbf{w}$ if and only if $a_1=a_2$ and $b_1=b_2$.

$\vec{i}=\langle 1,0\rangle; \vec{j}=\langle 0,1\rangle; \vec{v}=\langle a,b\rangle=a\langle 1,0\rangle+b\langle 0,1\rangle=a\vec{i}+b\vec{j}$

$$\mathbf{v}+\mathbf{w}=(a_1+a_2)\mathbf{i}+(b_1+b_2)\mathbf{j}=\langle a_1+a_2, b_1+b_2\rangle$$
$$\mathbf{v}-\mathbf{w}=(a_1-a_2)\mathbf{i}+(b_1-b_2)\mathbf{j}=\langle a_1-a_2, b_1-b_2\rangle$$
$$\alpha\mathbf{v}=(\alpha a_1)\mathbf{i}+(\alpha b_1)\mathbf{j}=\langle \alpha a_1, \alpha b_1\rangle$$
$$\|\mathbf{v}\|=\sqrt{a_1^2+b_1^2}$$

Example 3. (a) $\langle 8,3\rangle$ **(b)** $\langle 4,-7\rangle$

Example 4. (a) $\langle 30,-10\rangle$ **(b)** $\langle 14,-16\rangle$ **(c)** $2\sqrt{10}$

Example 5. (a) $6\vec{i}+9\vec{j}-6\vec{k}$ **(b)** $-5\vec{i}+18\vec{j}-19\vec{k}$ **(c)** $\sqrt{17}$

p. 256. 1; $\mathbf{u}=\dfrac{\vec{v}}{\|\vec{v}\|}$

Example 6. $\vec{u}=\dfrac{12}{13}\vec{i}-\dfrac{5}{13}\vec{j}$; find the magnitude of the unit vector to confirm that it is 1.

Exploration 4. magnitude; direction; $45\sqrt{3}; 45; \vec{v}=90\left[\cos 30°\vec{i}+\sin 30°\vec{j}\right]=45\sqrt{3}\vec{i}+45\vec{j}$;

≈77.94 mph; 45 mph

p. 257. $\cos\alpha;\sin\alpha$

Example 7. (a) $\vec{v}_a=-275\vec{i}-275\vec{j};\vec{v}_w=100\vec{i}$ **(b)** $\vec{v}_g=\left(100-275\sqrt{2}\right)\vec{i}-275\sqrt{2}\vec{j}$

(c) ≈484.5 mph; S36.6°W

Example 8. ≈732.1 lbs; ≈896.6 lbs

Section 9.5: The Dot Product

p. 258. $a_1a_2+b_1b_2$; scalar

Example 1. (a) 34 **(b)** 29

p. 258. **Commutative Property:** $\mathbf{u}\cdot\mathbf{v}=\mathbf{v}\cdot\mathbf{u}$

Distributive Property: $\mathbf{u}\cdot(\mathbf{v}+\mathbf{w})=\mathbf{u}\cdot\mathbf{v}+\mathbf{u}\cdot\mathbf{w}$

$$\begin{aligned}\mathbf{v}\cdot\mathbf{v}&=(a\mathbf{i}+b\mathbf{j})\cdot(a\mathbf{i}+b\mathbf{j})\\&=a^2+b^2\\&=\|\mathbf{v}\|^2\end{aligned}$$

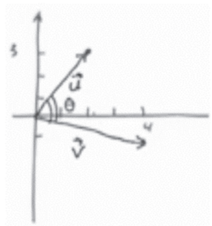
$\mathbf{0}\cdot\mathbf{v}=0$;

$\mathbf{v}\cdot\mathbf{w}=a_1a_2+b_1b_2+c_1c_2$

Example 2. (a) -5 **(b)** -5 **(c)** 49 **(d)** 35 **(e)** 7 **(f)** $\sqrt{35}$

Exploration 1.

$$\begin{aligned}\|\vec{u}-\vec{v}\|^2&=\|\vec{u}\|^2+\|\vec{v}\|^2-2\|\vec{u}\|\|\vec{v}\|\cos\theta\\(\vec{u}-\vec{v})\cdot(\vec{u}-\vec{v})&=\vec{u}\cdot\vec{u}+\vec{v}\cdot\vec{v}-2\|\vec{u}\|\|\vec{v}\|\cos\theta\\\vec{u}\cdot\vec{u}-\vec{u}\cdot\vec{v}-\vec{u}\cdot\vec{v}+\vec{v}\cdot\vec{v}&=\vec{u}\cdot\vec{u}+\vec{v}\cdot\vec{v}-2\|\vec{u}\|\|\vec{v}\|\cos\theta\\-2\vec{u}\cdot\vec{v}&=-2\|\vec{u}\|\|\vec{v}\|\cos\theta\\\cos\theta&=\frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\|\vec{v}\|}\end{aligned}$$

Example 3. (a)  **(b)** ≈70.3°

Example 4. (a) Vectors form line **(b)** 180° **(c)** $\vec{v} = -2\vec{w}$

Exploration 2. 0; $0 = \vec{v} \cdot \vec{w}$; 0

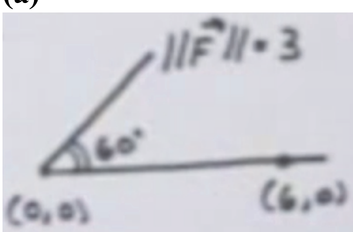
Example 5. (a) Vectors form a right angle **(b)** 0

p. 261. $\mathbf{v}_1 = \dfrac{\mathbf{v} \cdot \mathbf{w}}{\|\mathbf{w}\|^2}\mathbf{w}$; $\mathbf{v}_2 = \mathbf{v} - \mathbf{v}_1$

Example 6. $\mathbf{v}_1 = 2\mathbf{i} + \mathbf{j}$; $\mathbf{v}_2 = -\mathbf{i} + \mathbf{j}$

Example 7. $W = \vec{F} \cdot \vec{d}$

(a)



(b) $\vec{F} = \frac{3}{2}\vec{i} + \frac{3\sqrt{3}}{2}\vec{j}$

(c) $\vec{d} = 6\vec{i}$

(d) 9 foot-lbs.

Section 9.6: Vectors in Space

Exploration 1. $x; y; z; xy; xz; yz$

p. 257. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Example 1. $\sqrt{86}$

p. 263. $\mathbf{v} = (x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j} + (z_2 - z_1)\mathbf{k}$

Example 2. $\mathbf{v} = 2\mathbf{i} + 4\mathbf{j} + \mathbf{k}$

p. 263.

- $\mathbf{v} = \mathbf{w}$ if and only if $a_1 = a_2, b_1 = b_2$, and $c_1 = c_2$
- $\mathbf{v} + \mathbf{w} = (a_1 + a_2)\mathbf{i} + (b_1 + b_2)\mathbf{j} + (c_1 + c_2)\mathbf{k}$
- $\mathbf{v} - \mathbf{w} = (a_1 - a_2)\mathbf{i} + (b_1 - b_2)\mathbf{j} + (c_1 - c_2)\mathbf{k}$
- $\alpha\mathbf{v} = (\alpha a_1)\mathbf{i} + (\alpha b_1)\mathbf{j} + (\alpha c_1)\mathbf{k}$
- $\|\mathbf{v}\| = \sqrt{a_1^2 + b_1^2 + c_1^2}$

Example 3. **(a)** $\mathbf{i} + 3\mathbf{j} - \mathbf{k}$ **(b)** $5\mathbf{i} + 7\mathbf{j} - 3\mathbf{k}$ **(c)** $9\mathbf{i} + 15\mathbf{j} - 6\mathbf{k}$ **(d)** $12\mathbf{i} + 16\mathbf{j} - 7\mathbf{k}$
Example 4. 7
Example 5. $-1\mathbf{j} - 2\mathbf{k}$
p. 264.

$$\mathbf{u} = \frac{\mathbf{v}}{\|\mathbf{v}\|}$$

Example 6. $\frac{3}{7}\vec{i} - \frac{6}{7}\vec{j} - \frac{2}{7}\vec{k}$

p. 265. $\mathbf{v} \cdot \mathbf{w} = a_1a_2 + b_1b_2 + c_1c_2$

Example 7. **(a)** -13 **(b)** -13 **(c)** 29 **(d)** 35 **(e)** $\sqrt{29}$ **(f)** $\sqrt{35}$
p. 265.

Commutative Property

$$\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$$

Distributive Property

$$\mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) = \mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w}$$

$$\mathbf{v} \cdot \mathbf{v} = \|\mathbf{v}\|^2$$
$$\mathbf{0} \cdot \mathbf{v} = 0$$

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|}$$

Example 8. 166.7°
p. 266.

α = the angle between $\mathbf{v}$ and $\mathbf{i}$, the positive x-axis, $0 \le \alpha \le \pi$
β = the angle between $\mathbf{v}$ and $\mathbf{j}$, the positive y-axis, $0 \le \beta \le \pi$
γ = the angle between $\mathbf{v}$ and $\mathbf{k}$, the positive z-axis, $0 \le \gamma \le \pi$

- $\cos\alpha = \dfrac{a}{\sqrt{a^2+b^2+c^2}} = \dfrac{a}{\|\mathbf{v}\|}$
- $\cos\beta = \dfrac{b}{\sqrt{a^2+b^2+c^2}} = \dfrac{b}{\|\mathbf{v}\|}$
- $\cos\gamma = \dfrac{c}{\sqrt{a^2+b^2+c^2}} = \dfrac{c}{\|\mathbf{v}\|}$

Example 9. $\alpha \approx 72.7°; \beta \approx 126.6°; \gamma \approx 41.8°$

p. 266. $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

Example 10. $\frac{\pi}{2}$

Section 9.7: The Cross Product

p. 267. $\mathbf{v}\times\mathbf{w} = (b_1c_2 - b_2c_1)\mathbf{i} - (a_1c_2 - a_2c_1)\mathbf{j} + (a_1b_2 - a_2b_1)\mathbf{k}$

Example 1. $\mathbf{i} + \mathbf{j} + 4\mathbf{k}$

Example 2. (a) -25 **(b)** $-4A + 11B - 6C$

$$\mathbf{v}\times\mathbf{w} = (b_1c_2 - b_2c_1)\mathbf{i} - (a_1c_2 - a_2c_1)\mathbf{j} + (a_1b_2 - a_2b_1)\mathbf{k}$$

Example 3. (a) -17**i** - 18 **j** + 10**k** **(b)** 17**i** + 18 **j** - 10**k** **(c) 0 (d) 0 (e)** negatives of one another **(f)** cross product of a vector and itself is the zero vector

p. 268.

- $\mathbf{u}\times\mathbf{u} = \mathbf{0}$
- $\mathbf{u}\times\mathbf{v} = -(\mathbf{v}\times\mathbf{u})$
- $\alpha(\mathbf{u}\times\mathbf{v}) = (\alpha\mathbf{u})\times\mathbf{v} = \mathbf{u}\times(\alpha\mathbf{v})$
- $\mathbf{u}\times(\mathbf{v}+\mathbf{w}) = (\mathbf{u}\times\mathbf{v}) + (\mathbf{u}\times\mathbf{w})$

- $\mathbf{u}\times\mathbf{v}$ is orthogonal to both $\mathbf{u}$ and $\mathbf{v}$.
- $\|\mathbf{u}\times\mathbf{v}\| = \|\mathbf{u}\|\,\|\mathbf{v}\|\sin\theta$, where θ is the angle between $\mathbf{u}$ and $\mathbf{v}$.
- $\|\mathbf{u}\times\mathbf{v}\|$ is the area of the parallelogram having $\mathbf{u} \neq \mathbf{0}$ and $\mathbf{v} \neq \mathbf{0}$ as adjacent sides.
- $\mathbf{u}\times\mathbf{v} = \mathbf{0}$ if and only if $\mathbf{u}$ and $\mathbf{v}$ are parallel.

Example 4. (a) -9**i** - 7 **j** - 3**k** **(b)** zero

Example 5. $2\sqrt{2393} \approx 97.8$ square units

CHAPTER 10: Analytic Geometry

Section 10.1: Conics

p. 270. right circular cone; plane **(a)** perpendicular **(b)** tilted slightly **(c)** parallel **(d)** nappes; circle; ellipse; parabola; hyperbola

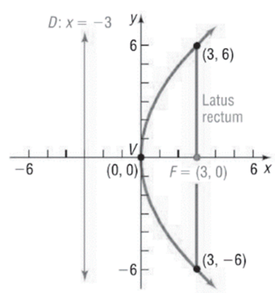
Section 10.2: The Parabola

p. 271. focus; directrix; $d(F,P) = d(P,D)$; symmetry; vertex

Exploration 1.

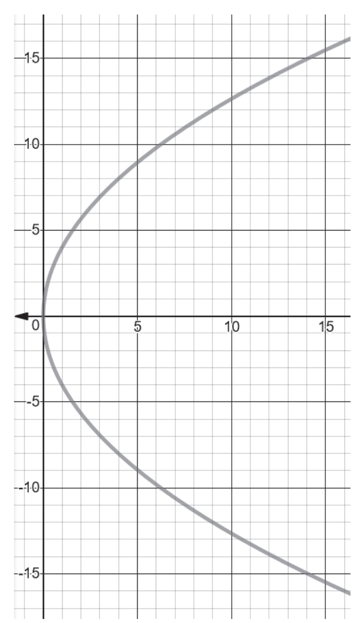
$$d(F,P) = d(P,D)$$
$$\sqrt{(x-a)^2+(y-0)^2} = \sqrt{(-a-x)^2+(y-y)^2}$$
$$(x-a)^2 + y^2 = (-a-x)^2$$
$$x^2 - 2ax + a^2 + y^2 = x^2 + 2ax + a^2$$
$$y^2 = 4ax$$

Example 1. (a) $y^2 = 12x$ **(b)**



p. 272. $y^2 = 4ax$

Example 2. (a) (0, 0) **(b)** (4, 0) **(c)** $x = -4$ **(d)** (4, 8); (4, -8) **(e)**



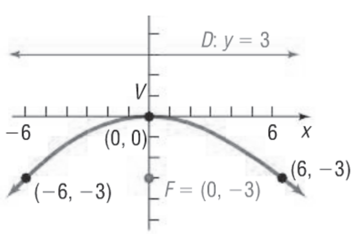
p. 273. $y^2 = 4ax; y^2 = -4ax; x^2 = 4ay; x^2 = -4ay$

Equations of a Parabola: Vertex at (0, 0); Focus on an Axis; $a > 0$

Vertex	Focus	Directrix	Equation	Description
(0, 0)	$(a, 0)$	$x = -a$	$y^2 = 4ax$	Axis of symmetry is the x-axis, the parabola opens right
(0, 0)	$(-a, 0)$	$x = a$	$y^2 = -4ax$	Axis of symmetry is the x-axis, the parabola opens left
(0, 0)	$(0, a)$	$y = -a$	$x^2 = 4ay$	Axis of symmetry is the y-axis, the parabola opens up (is concave up)
(0, 0)	$(0, -a)$	$y = a$	$x^2 = -4ay$	Axis of symmetry is the y-axis, the parabola opens down (is concave down)

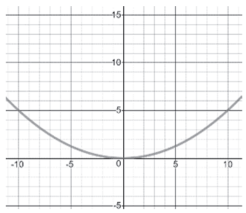
Example 3. (a) (0, 0) **(b)** (0,-3) **(c)** $y = 3$ **(d)** (-6, -3); (6, -3)

(e)



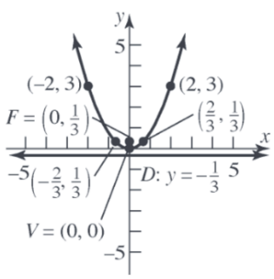
Example 4. (a) $x^2 = 20y$

(b)



Example 5. (a) $x^2 = \frac{4}{3}y$

(b)



(c) $\left(0, \frac{1}{3}\right); y = -\frac{1}{3}$

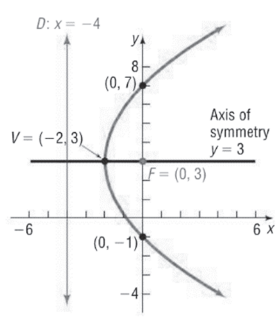
(d) confirms

p. 275. $(y-k)^2 = 4a(x-h); (y-k)^2 = -4a(x-h); (x-k)^2 = 4a(y-h); (x-k)^2 = -4a(y-h)$

Equations of a Parabola: Vertex at (h, k); Axis of Symmetry Parallel to a Coordinate Axis; $a > 0$

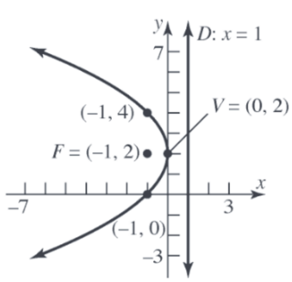
Vertex	Focus	Directrix	Equation	Description
(h, k)	$(h+a, k)$	$x = h - a$	$(y-k)^2 = 4a(x-h)$	Axis of symmetry is parallel to the x-axis, the parabola opens right
(h, k)	$(h-a, k)$	$x = h + a$	$(y-k)^2 = -4a(x-h)$	Axis of symmetry is parallel to the x-axis, the parabola opens left
(h, k)	$(h, k+a)$	$y = k - a$	$(x-h)^2 = 4a(y-k)$	Axis of symmetry is parallel to the y-axis, the parabola opens up (is concave up)
(h, k)	$(h, k-a)$	$y = k + a$	$(x-h)^2 = -4a(y-k)$	Axis of symmetry is parallel to the y-axis, the parabola opens down (is concave down)

Example 6. (a) $(y-3)^2 = 8(x+2)$ **(b)**



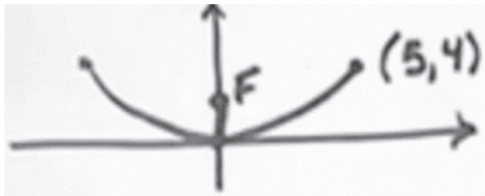
Example 7. (a) $(y-2)^2 = -4x$ **(b)** (0, 2) **(c)** (-1, 2) **(d)** $x = 1$

(e)



p. 276. Answers will vary.

Example 8. (a)



(b) 1.5625 feet, or 18.75 inches from the base of the dish

Section 10.3: The Ellipse

p. 277. foci
Exploration 1. x – axis; origin;

$$d(F_1,P)+d(F_2,P)=2a$$
$$\sqrt{(x+c)^2+y^2}+\sqrt{(x-c)^2+y^2}=2a$$
$$\sqrt{(x+c)^2+y^2}=2a-\sqrt{(x-c)^2+y^2}$$

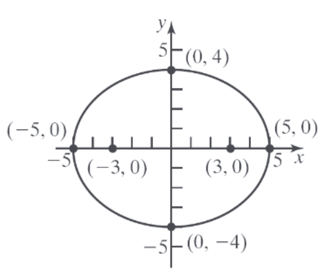
$$(x+c)^2+y^2=4a^2-4a\sqrt{(x-c)^2+y^2}+(x-c)^2+y^2$$
$$x^2+2cx+c^2+y^2=4a^2-4a\sqrt{(x-c)^2+y^2}+x^2-2cx+c^2+y^2$$
$$4cx-4a^2=-4a\sqrt{(x-c)^2+y^2}$$
$$cx-a^2=-a\sqrt{(x-c)^2+y^2}$$
$$(cx-a^2)^2=a^2[(x-c)^2+y^2]$$
$$c^2x^2-2a^2cx+a^4=a^2(x^2-2cx+c^2+y^2)$$
$$(c^2-a^2)x^2-a^2y^2=a^2c^2-a^4$$
$$(a^2-c^2)x^2+a^2y^2=a^2(a^2-c^2)$$

To make notation easier, let $b^2=a^2-c^2$

$$b^2x^2+a^2y^2=a^2b^2 \text{(Substitute)}$$
$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \text{(Divide by } a^2b^2 \text{ and simplify)}$$

p. 278. $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$

Example 1. (a) major axis is the x – axis since the larger denominator is of the x - term



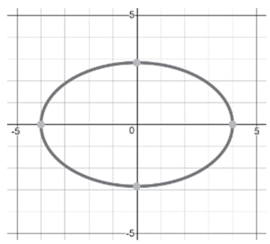
(b) $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$; the distance from the center to one of the vertices; 5; the distance from the center to a foci; 3; $b^2=a^2-c^2$; 4

(c) $\dfrac{x^2}{25}+\dfrac{y^2}{16}=1$

(d – e) Answers will vary.

Example 2. (a) (-4, 0) (4, 0) **(b)** $\left(-2\sqrt{2},0\right)\left(2\sqrt{2},0\right)$

(c)

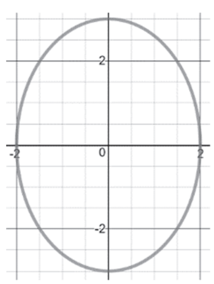


(d) major axis is the x – axis since the larger denominator is of the x - term

p. 279. $\dfrac{x^2}{b^2}+\dfrac{y^2}{a^2}=1$

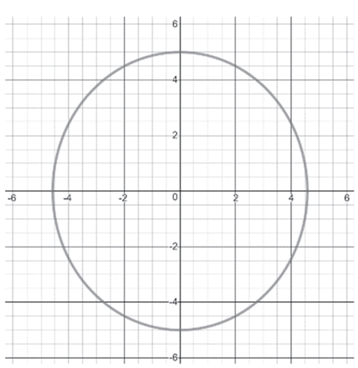
Example 3. (a) $\dfrac{x^2}{4}+\dfrac{y^2}{9}=1$ **(b)** $\left(0,\pm 3\right)$ **(c)** $\left(0,\pm\sqrt{5}\right)$

(d)



(e) major axis is the y – axis since the larger denominator is of the y - term

Example 4. (a) major axis is the y – axis since it is longer



(b) $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$; the distance from the center to one of the vertices; 5; the distance from the center to a foci; 2; $b^2=a^2-c^2$; $\sqrt{21}$

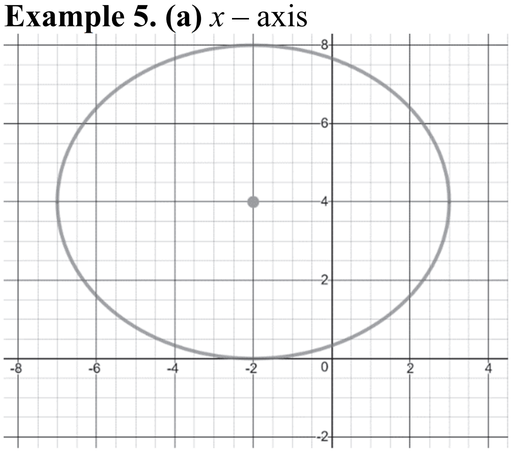
(c) $\frac{x^2}{21}+\frac{y^2}{25}=1$

Exploration 2. $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$; $\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$

Equations of an Ellipse: Center at (*h*, *k*); Major Axis Parallel to a Coordinate Axis

Center	Major Axis	Foci	Vertices	Equation
(h, k)	Parallel to the x-axis	$(h+c, k)$	$(h+a, k)$	$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$
		$(h-c, k)$	$(h-a, k)$	$a>b>0$ and $b^2=a^2-c^2$
(h, k)	Parallel to the y-axis	$(h, k+c)$	$(h, k+a)$	$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$
		$(h, k-c)$	$(h, k-a)$	$a>b>0$ and $b^2=a^2-c^2$

Example 5. (a) $x-$axis

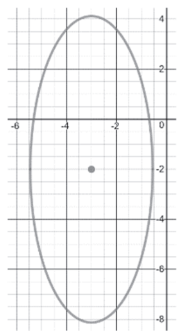


(b) $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$; the distance from the center to one of the vertices; 5; the distance from the center to a foci; 3; $b^2=a^2-c^2$; 4

(c) $\frac{(x+2)^2}{25}+\frac{(y-4)^2}{16}=1$

Example 6. (a) $\frac{(x+3)^2}{6}+\frac{(y+2)^2}{37.5}=1$ **(b)** (-3, -2) **(c)** $\left(-3,-2\pm\sqrt{31.5}\right)$ **(d)** $\left(-3,-2\pm\sqrt{37.5}\right)$

(e) y – axis **(f)**



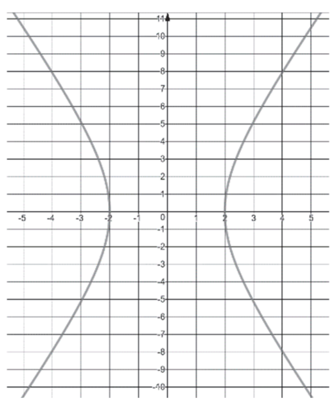
Example 7. 128 feet

Section 10.4: The Hyperbola

p. 284. Foci; transverse; conjugate

Exploration 1. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$; the x^2 term is first

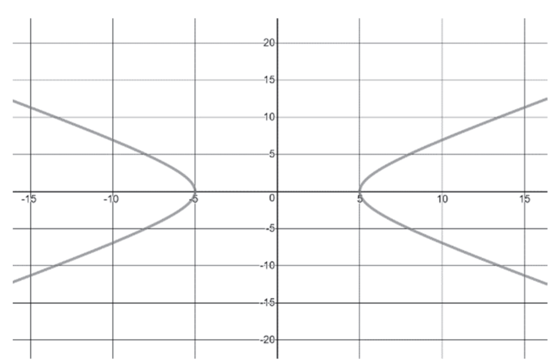
Example 1. (a) x – axis since the x^2 term is first



(b) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$; the distance from the center to one of the vertices; 2; the distance from the center to a foci; 5; $b^2 = c^2 - a^2$; $\sqrt{21}$

(c) $\frac{x^2}{4} - \frac{y^2}{21} = 1$

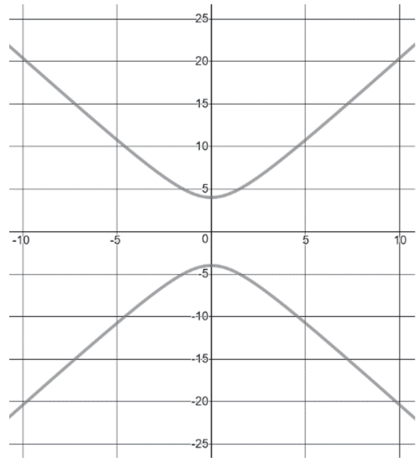
Example 2. (a) $(\pm 5, 0)$ **(b)** $(\pm\sqrt{41}, 0)$ **(c)**



(d) x – axis since the x^2 term is first

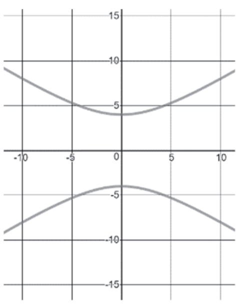
p. 286. $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

Example 3. (a) $\frac{y^2}{16}-\frac{x^2}{4}=1$ **(b)** $(0,\pm 4)$ **(c)** $\left(0,\pm 2\sqrt{5}\right)$ **(d)**



(e) y – axis since the y^2 term is first

Example 4. (a) y – axis since the foci lie on this axis



(b) $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$; the distance from the center to one of the vertices; 4; the distance from the center to a foci; 7; $b^2=c^2-a^2$; $\sqrt{33}$

(c) $\frac{y^2}{16}-\frac{x^2}{33}=1$

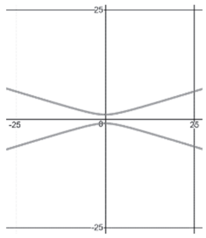
Exploration 2. The asymptote approaches $y=\pm\frac{b}{a}x$

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$
$$\frac{y^2}{b^2}=\frac{x^2}{a^2}-1$$
$$y^2=\frac{b^2}{a^2}x^2-b^2$$
$$y=\pm\sqrt{\frac{b^2}{a^2}x^2-b^2}$$

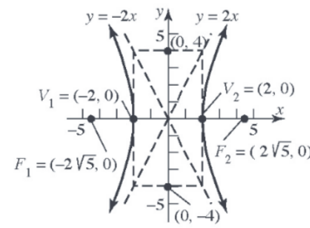
p. 288. $y=\frac{b}{a}x$ and $y=-\frac{b}{a}x$; $y=\frac{a}{b}x$ and $y=-\frac{a}{b}x$

Example 5. (a) (0, 1) (0, -1) **(b)** $\left(0,\pm\sqrt{17}\right)$ **(c)** $y=\pm\frac{1}{4}x$

(d)



(e) y – axis since the y^2 term is first



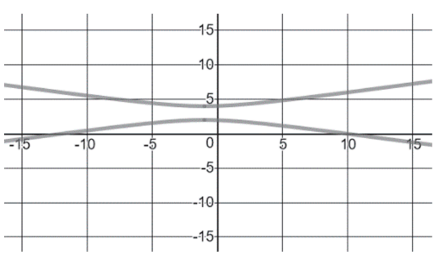
Example 6. (a) $\frac{x^2}{4}-\frac{y^2}{16}=1$ **(b)** $(\pm 2, 0)$ **(c)** $(\pm 2\sqrt{5}, 0)$ **(d)** $y=\pm 2x$ **(e)**

Exploration 3. $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1; \frac{(x-k)^2}{a^2}-\frac{(y-h)^2}{b^2}=1$

Equations of a Hyperbola: Center at (h, k); Transverse Axis Parallel to a Coordinate Axis

Center	Transverse Axis	Foci	Vertices	Equation	Asymptotes
(h, k)	Parallel to the x-axis	$(h \pm c, k)$	$(h \pm a, k)$	$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1,\quad b^2=c^2-a^2$	$y-k=\pm\frac{b}{a}(x-h)$
(h, k)	Parallel to the y-axis	$(h, k \pm c)$	$(h, k \pm a)$	$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1,\quad b^2=c^2-a^2$	$y-k=\pm\frac{a}{b}(x-h)$

Example 7(a) y – axis since the key points are parallel to this axis



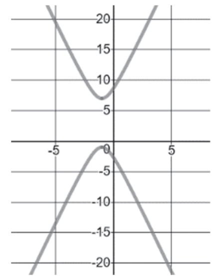
(b) $\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$; the distance from the center to a vertex; 1; the distance from the center to a focus; 4; $b^2=c^2-a^2$; $\sqrt{15}$

(c) $\frac{(y-3)^2}{1}-\frac{(x+1)^2}{16}=1$

(d) $y-3=\pm\frac{1}{\sqrt{15}}(x+1)$

Example 8. (a) $\frac{(y-3)^2}{16}-\frac{(x+2)^2}{1}=1$ **(b)** y – axis **(c)** (-2, 3) **(d)** $\left(-2, 3\pm\sqrt{17}\right)$

(e) $(-2,7)(-2,-1)$ **(f)** $y-3=\pm 4(x+2)$ **(g)**



Example 9. 202,202 feet north of you

Section 10.5: Rotation of Axes; General Form of a Conic

p. 292. $Ax^2+Cy^2+Dx+Ey+F; AC; AC; AC$

Example 1. (a) parabola **(b)** ellipse (or circle) **(c)** hyperbola

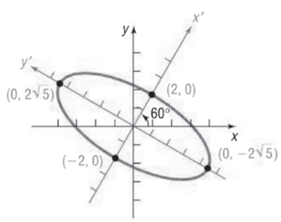
p. 292. $x = x'\cos\theta - y'\sin\theta; y = x'\sin\theta + y'\cos\theta$

Example 2. $\frac{x'^2}{4}-\frac{y'^2}{20}=1$; ellipse with center at (0, 0) and major axis along the x' axis.

p. 293. $\frac{A-C}{B}$

Example 3. (a) $\frac{x'^2}{4}-\frac{y'^2}{20}=1$

(b)



p. 293. $Ax^2+Bxy+Cy^2+Dx+Ey+F; B^2-4AC; B^2-4AC; B^2-4AC$

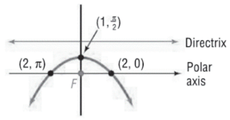
Example 4. Since $B^2-4AC = 17 > 0$ the graph is a hyperbola

Section 10.6: Polar Equations of Conics

p. 294. directrix; focus; $\frac{d(F,P)}{d(D,P)}=e$;

$\perp$, left; $\perp$, right; $\parallel$, above; $\parallel$, below;
parabola; $\perp$; ellipse; $\perp$; hyperbola; $\perp$

Example 1. e = 1 and p = 2; parabola with focus at the pole; directrix is parallel to the polar axis 2 units above the pole; axis of symmetry is perpendicular to the polar axis. Vertex is $\left(1,\frac{\pi}{2}\right)$



Example 2. $e = \frac{1}{8}; p = 24$; ellipse; major axis perpendicular to the directrix; directrix is parallel to the polar axis 24 units below the pole.

Example 3. $e = \frac{1}{2}; p = \frac{3}{2}$; ellipse; major axis perpendicular to the directrix; directrix is perpendicular to the polar axis $\frac{3}{2}$ units left of the pole.

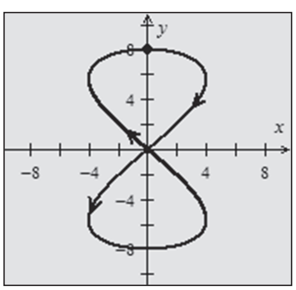
Example 4. $y^2 + 2x - 1 = 0$

Section 10.7: Plane Curves and Parametric Equations

p. 296. $x(t); y(t); (x(t), y(t))$

Exploration 1.

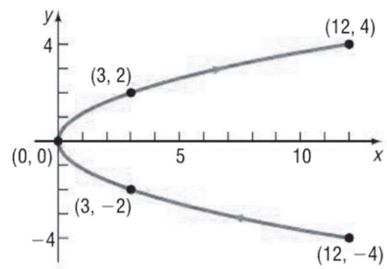
t	*x*	*y*	*(x, y)*
0	0	8	(0, 8)
1	4	5.66	(4, 5.66)
2	0	0	(0, 0)
3	-4	-5.66	(-4, -5.66)
4	0	-8	(0, -8)
5	4	5.66	(4, 5.66)
6	0	0	(0, 0)
7	-4	5.66	(-4, 5.66)
8	0	8	(0, 8)



vertical; time

Example 1.

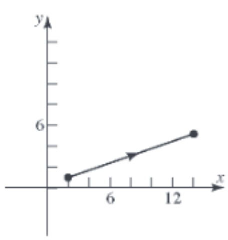
t	x	y	(x, y)
−2	12	−4	(12, −4)
−1	3	−2	(3, −2)
0	0	0	(0, 0)
1	3	2	(3, 2)
2	12	4	(12, 4)



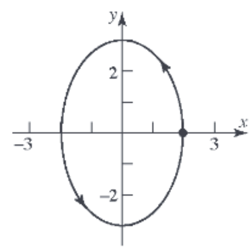
Example 2. (a) parabolic **(b)** $x=\frac{3y^2}{4}$ **(c)** $y=\pm\sqrt{\frac{4}{3}x}; 0; 12$ **(d)** In addition to showing the path of the curve, a system of parametric equations shows the location of each point at a certain time.

Example 3.

(a)

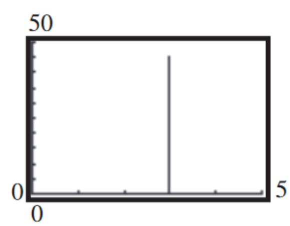


(b)



Example 4. (a) $x-3y+1=0$ **(b)** $\frac{x^2}{4}+\frac{y^2}{9}=1$

Example 5. (a) $y(t)=-16t^2+50t+6; x(t)=0$ **(b)** ≈ 3.24 seconds **(c)** At 1.5625 seconds the ball is at its maximum height of 45.0625 feet **(d)**



Example 6.
$x=t^{3/2}, y=t; x=\sqrt{t}, y=\sqrt[3]{t}$

Example 7. $x=2\cos(\pi t), y=-3\sin(\pi t), 0\le t\le 2$

CHAPTER 11: Systems of Equations and Inequalities

Section 11.1: Systems of Linear Equations: Substitution and Elimination

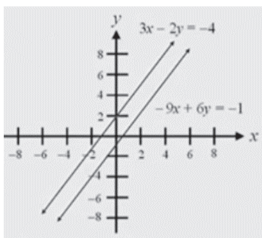
p. 300. two; solutions
Example 1. (a) $y=2x-13$ **(b)** $-4x-9(2x-13)=7$ **(c)** $x=5$ **(d)** $y=-3$
(e) $2\cdot 5-(-3)=10+3=13; -4\cdot 5-9\cdot(-3)=-20+27=7$

Example 2. (a) $24\left(\frac{3}{2}x-\frac{y}{8}\right)=24(-1); 36x-3y=-4$ **(b)** $52x=-52$ **(c)** $x=-1$ **(d)** $y=-4$

(e) $\frac{3}{2}\cdot(-1)-\frac{-4}{8}=-\frac{3}{2}+\frac{4}{8}=-1; 16\cdot(-1)+3\cdot(-4)=-16-12=-28$

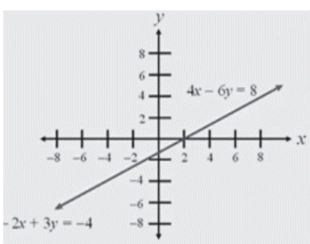
Exploration 1. $C(m_A)=0.065m+24{,}200; C(m_B)=0.09m+16{,}800$; $m=296{,}000; C=43{,}440$;
The substitution method was used because both functions equal cost; after 296,000 miles driven, the total cost of the cars is the same. Knowing this may help me decide which car to buy depending on how many miles I plan to drive the car

Example 3. (a) (b) parallel; no solution; { }; ∅ **(c)** solve for *y* to see that the slopes are the same



Example 4. (a) Multiply second equation by -3, then add to get 0 = -9 **(b)** no; parallel

Example 5. (a) (b) coincide; consistent; dependent; infinitely; $\{(x,y)\,|\,4x-6y=8\}$



Example 6. (a) Multiply the first equation by 3 and the second equation by 2 to end up with 0 = 0 **(b)** $\{(x,y)\,|\,6x-4y=8\}$; the same

Exploration 2. same; A consistent system with independent equations; An inconsistent system; A consistent system with dependent equations; plane in

Example 7. (-3, 0, 1) is a solution

Example 8. The solution is (2, -1, 1)

Example 9. Multiply the first equation by 2 and add to the second equation to get 0 = 2; false; inconsistent; { }

Example 10. End up with 0 = 0; true; dependent;
$\{(x,y,z)\,|\,x=5z+2, y=-3z+1, z \text{ is any real number}\}$

Section 11.2: Systems of Linear Equations: Matrices

Exploration 1. entry; row; column; row 3, column 5; augmented

Example 1. (a) $\left[\begin{array}{cc|c} 3 & -2 & 3 \\ -2 & 1 & -2 \end{array}\right]$ **(b)** $\left[\begin{array}{ccc|c} 3 & -2 & 0 & -5 \\ -2 & 0 & 4 & -2 \\ 1 & 4 & -7 & 0 \end{array}\right]$

Example 2. (a) $\begin{cases} -2x+y=3 \\ x+y=-2 \end{cases}$ **(b)** $\begin{cases} 3x-2y+5z=3 \\ -2x+y+4z=-2 \\ x+4y-7z=1 \end{cases}$

Exploration 2. two; a nonzero multiple; sum; some other row

two; flip-flopping any two equations in a system

a nonzero multiple; multiply both sides of an equation by a nonzero constant

sum; some other row; multiply a row by a number, then add that row to another row to replace a row.

Example 3. (a) The *R* means that this is a new row and the subscript tells you which row to replace**(b)** take -3 times "old" row 1 plus "old" row 2 **(b)** $\left[\begin{array}{cc|c} 1 & -2 & 2 \\ 0 & 1 & 3 \end{array}\right]$

Example 4. (a) Multiply the entries in row 1 by -2 and then add the result to the entries in row 2, or $R_2 = -2r_1 + r_2$, $\left[\begin{array}{ccc|c} 1 & 2 & 3 & -7 \\ 2 & 5 & -1 & 0 \\ -3 & 6 & 2 & -5 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & -7 \\ 0 & 1 & -7 & 14 \\ -3 & 6 & 2 & -5 \end{array}\right]$

(b) $R_3 = 3r_1 + r_3$, $\left[\begin{array}{ccc|c} 1 & 2 & 3 & -7 \\ 2 & 5 & -1 & 0 \\ -3 & 6 & 2 & -5 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & -7 \\ 2 & 5 & -1 & 0 \\ -3 & 6 & 2 & -5 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & -7 \\ 0 & 1 & -7 & 14 \\ 0 & 12 & 11 & -26 \end{array}\right]$

Exploration 3. 1, 1; a 1, right; 0's; $\left[\begin{array}{ccc|c} 1 & 5 & -6 & 3 \\ 0 & 1 & 3 & 12 \\ 0 & 0 & 1 & -3 \end{array}\right]$; the last row has only one variable and allows us to solve for it; the second row has two variables and the variable from the last row can be substituted in to solve for the second variable; the two solved variables can be substituted into the first row to finish the solving of the system of linear equations

Example 5. (a) $\left[\begin{array}{cc|c} 3 & 11 & 13 \\ 1 & 5 & 7 \end{array}\right]$ **(b)** $\left[\begin{array}{cc|c} 1 & 5 & 7 \\ 3 & 11 & 13 \end{array}\right]$ **(c)** interchange rows 1 and 2 **(d)** $R_2 = -3r_1 + r_2$; $\left[\begin{array}{cc|c} 1 & 5 & 7 \\ 0 & -4 & -3 \end{array}\right]$ **(e)** $R_2 = -\frac{1}{4}r_2$; $\left[\begin{array}{cc|c} 1 & 5 & 7 \\ 0 & 1 & 2 \end{array}\right]$ **(f)** $\begin{cases} x+5y=7 \\ y=-2 \end{cases}$; (-3, 2)

p. 310.

Matrix Method for Solving a System of Linear Equations (Row Echelon Form)

STEP 1: Write the augmented matrix that represents the system.

STEP 2: Use row operations to obtain 1 in row 1, column 1.

STEP 3: Use row operations that leave row 1 unchanged, but change the entries in column 1 below row 1 to 0's.

STEP 4: Use row operations to obtain 1 in row 2, column 2, but leave the entries in columns to the left unchanged. If it is impossible to place 1 in row 2, column 2, place 1 in row 2, column 3. Once the 1 is in place, use row operations to obtain 0's below it. (Place any rows that contain only 0's on the left side of the vertical bar, at the bottom of the matrix.)

STEP 5: Now repeat Step 4 to obtain 1 in the next row, but one column to the right. Continue until the bottom row or the vertical bar is reached.

STEP 6: The matrix that results is the row echelon form of the augmented matrix. Analyze the system of equations corresponding to it to solve the original system.

Example 6. (2, -2, 1)

Example 7. $\left(-\frac{7}{5}z-\frac{36}{5},\frac{1}{5}z-\frac{22}{5},z\right)$

Example 8. $\varnothing$

Section 11.3: Systems of Linear Equations: Determinants

p. 312. substitution and elimination; matrices; $D=\begin{vmatrix} a & b \\ c & d \end{vmatrix}$; $ad-bc$; $D=\begin{vmatrix} a & b \\ c & d \end{vmatrix}=ad-bc$

Example 1. (a) $\begin{vmatrix} -2 & 3 \\ 4 & -1 \end{vmatrix}=-10$ **(b)** using graphing calculator to arrive at same result

$$x=\frac{\begin{vmatrix} s & b \\ t & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}} \qquad y=\frac{\begin{vmatrix} a & s \\ c & t \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}};\ 0$$

p. 313. $x=\frac{D_x}{D}$ $\qquad y=\frac{D_y}{D}$

Example 2. (a) (4, -2) **(b)** using graphing calculator to arrive at same result

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}=a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}-a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}+a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix};\ i\text{th};\ j\text{th}$$

Example 3. $M_{21}=-9$

p. 314. $A_{ij} = (-1)^{i+j} M_{ij}$

Example 4. (a) -1 **(b)** -1

$x = \dfrac{D_x}{D} \qquad y = \dfrac{D_y}{D} \qquad z = \dfrac{D_z}{D}$

Example 5. $(-2, 2, -1)$

Exploration 1. $\begin{vmatrix} -2 & 3 \\ 4 & -1 \end{vmatrix} = -10$; $\begin{vmatrix} 4 & -1 \\ -2 & 3 \end{vmatrix} = 10$;

$\begin{vmatrix} 1 & 2 & 1 \\ 1 & 2 & 1 \\ 2 & 6 & 7 \end{vmatrix} = 0$;

$\begin{vmatrix} -2 & 3 \\ 4 & -1 \end{vmatrix} = -10$; $\begin{vmatrix} -2k & 3k \\ 4 & -1 \end{vmatrix} = 2k - 12k = -10k = k\begin{vmatrix} -2 & 3 \\ 4 & -1 \end{vmatrix}$;

$\begin{vmatrix} -2 & 3 \\ 4 & -1 \end{vmatrix} = -10$; $\begin{vmatrix} -2 & 3 \\ 4 & -1 \end{vmatrix} \to \begin{vmatrix} -10 & 5 \\ 4 & -1 \end{vmatrix} = -10$

Section 11.4: Matrix Algebra

p. 317. same; same; two; corresponding

Example 1. (a) $\begin{bmatrix} -2 & -2 & 6 \\ 2 & 0 & -1 \end{bmatrix}$ **(b)** $\begin{bmatrix} 4 & -2 & -1 \\ -2 & -2 & 7 \end{bmatrix}$

$A + B = B + A$**;** $(A+B)+C = A+(B+C)$; additive

Example 2. (a) $4A = \begin{bmatrix} 12 & 4 & 20 \\ -8 & 0 & 24 \end{bmatrix}$ **(b)** $\frac{1}{3}\text{C} = \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}$ **(c)** $\begin{bmatrix} 1 & 1 & 15 \\ -22 & -2 & 24 \end{bmatrix}$

p. 318. 1 by n; $R=\begin{bmatrix} r_1 & r_2 & \ldots & \ldots & r_n \end{bmatrix}$; n by 1; $C=\begin{bmatrix} c_1 \\ c_2 \\ . \\ . \\ c_n \end{bmatrix}$;

$$RC=\begin{bmatrix} r_1 & r_2 & \ldots & \ldots & r_n \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \\ . \\ . \\ c_n \end{bmatrix}=r_1c_1+r_2c_2+\ldots+r_nc_n$$

Example 3. 16

p. 318. m by n; $i, j,$ ith, jth

Example 4. (a) the number of columns in matrix A is the same as the number of rows in matrix B **(b)** 2×2 **(c)** $\begin{bmatrix} 5 & 7 \\ -1 & 11 \end{bmatrix}$

Example 5. No **(a)** the number of columns in matrix B is the same as the number of rows in matrix A. **(b)** 3×3, the number of rows in first matrix is 3 and the number of columns in the second matrix is 3 **(c)** $BA=\begin{bmatrix} 6 & 12 & -2 \\ -3 & 14 & -4 \\ -9 & 10 & -4 \end{bmatrix}$

Example 6. (a) $\begin{bmatrix} 7 & 12 \\ -17 & -28 \end{bmatrix}$ **(b)** $\begin{bmatrix} -2 & -6 \\ -5 & -19 \end{bmatrix}$

p. 319. not; $A(BC)=(AB)C$; $A(B+C)=AB+AC$

1's, 0's, $I_2=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $I_3=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Example 7. (a) $\begin{bmatrix} 3 & -2 & 1 \\ 0 & 4 & -1 \end{bmatrix}$ **(b)** $\begin{bmatrix} 3 & -2 & 1 \\ 0 & 4 & -1 \end{bmatrix}$ **(c)** $\begin{bmatrix} 2 & 4 \\ -1 & 3 \\ -3 & 1 \end{bmatrix}$

p. 320. A, A; A

I_n

Example 8. $A \cdot A^{-1} = I_2; A^{-1} \cdot A = I_2$

p. 320. $[A \mid I_n]$; reduced row echelon; left, right

Example 9. $A^{-1} = \begin{bmatrix} \frac{7}{9} & -\frac{5}{9} & \frac{1}{9} \\ -\frac{2}{3} & \frac{1}{3} & \frac{1}{3} \\ -\frac{2}{9} & \frac{4}{9} & \frac{1}{9} \end{bmatrix}$

Example 10. The determinant of the matrix is 0, and thus is singular and has no inverse

Example 11. $\left(\frac{16}{9}, -\frac{5}{3}, -\frac{11}{9}\right)$

Section 11.5: Partial Fraction Decomposition

p. 322. $\frac{6x+16}{x^2+6x+8}$; reverse; four

$Q(x) = (x-a_1)(x-a_2)\ldots(x-a_n)$; $\frac{P(x)}{Q(x)} = \frac{A_1}{x-a_1} + \frac{A_2}{x-a_2} + \ldots + \frac{A_n}{x-a_n}$

Example 1. $\frac{x}{x^2-5x+6} = \frac{-2}{x-2} + \frac{3}{x-3}$

p. 323. $\frac{A_1}{x-a} + \frac{A_2}{(x-a)^2} + \ldots + \frac{A_n}{(x-a)^n}$

Example 2. $\frac{x+2}{x^3-2x^2+x} = \frac{2}{x} + \frac{-2}{x-1} + \frac{3}{(x-1)^2}$

p. 323. $\frac{Ax+B}{ax^2+bx+c}$

Example 3. $\frac{1}{(x+1)(x^2+4)} = \frac{\frac{1}{5}}{x+1} + \frac{-\frac{1}{5}x + \frac{1}{5}}{x^2+4}$

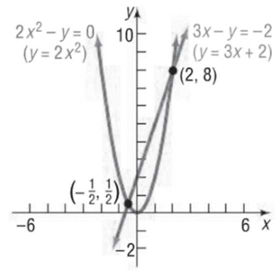
p. 324. $\frac{A_1x+B_1}{ax^2+bx+c} + \frac{A_2x+B_2}{(ax^2+bx+c)^2} + \ldots + \frac{A_nx+B_n}{(ax^2+bx+c)^n}$

Example 4. $\frac{x^2-5x-2}{(x^2+4)^2} = \frac{5}{x^2+4} + \frac{-5x-22}{(x^2+4)^2}$

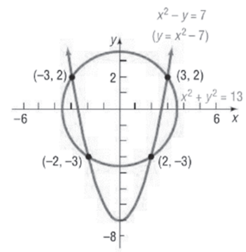
Section 11.6: Systems of Nonlinear Equations

p. 325. point(s) of intersection; substitution; elimination

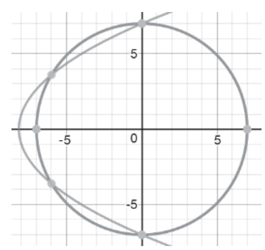
Example 1. (a) $\left(-\frac{1}{2},\frac{1}{2}\right),(2,8)$ **(b)**



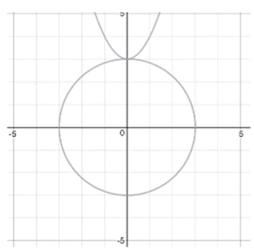
Example 2. (a)
$(-2,-3),(2,-3),(-3,2),(3,2)$ **(b)**



Example 3. (a) $(0,7),(0,-7),(-6,\sqrt{13}),(-6,-\sqrt{13})$ **(b)**



Example 4. (a) (0, 3) **(b)**

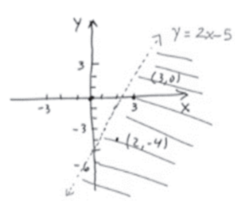


Example 5. ≈3.12 miles

Section 11.7: Systems of Inequalities

p. 328. graph; shaded

Example 1. (a) =; $y = 2x - 5$ **(b)** dashed; solid;

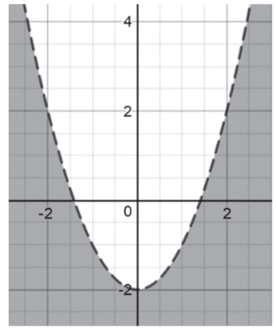


(c) Test point (0, 0) produces 0 > 5 which is false.

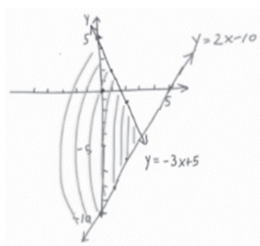
(d) all; none

(e) Answers will vary.

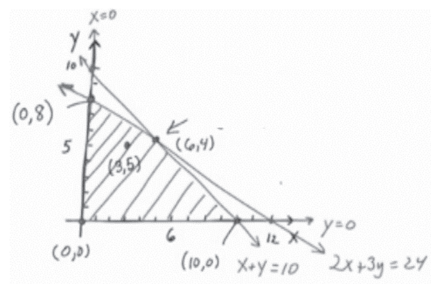
Example 2. =; dashes; solid; shade; do not shade;



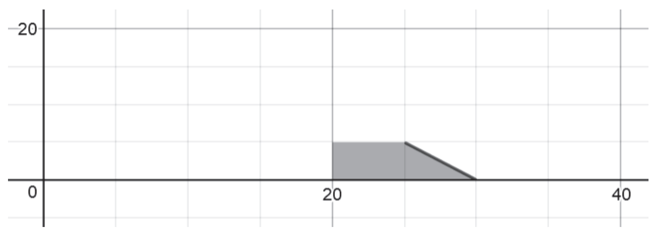
Example 3. shaded; solution set



Example 4.



Example 5. (a) $\begin{cases} x \geq 0 \\ y \geq 0 \\ x + y \leq 30 \\ x \geq 20 \\ y \leq 5 \end{cases}$ **(b)**

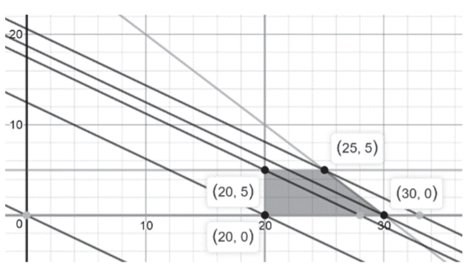


Section 11.8: Linear Programming

Example 1. (a) $I = 0.05x + 0.08y$; objective function **(b)** constraints

p. 331. $z = Ax + By$

Example 2.



p. 332. solution; at a corner point of the graph of the feasible points; at least one of them is located at a corner point of the graph of the feasible points; unique

Steps for Solving a Linear Programming Problem

Step 1: Assign symbols for the variables in the problem, and write an expression for the quantity to be maximized (or minimized). This expression is the objective function.

Step 2: Write all the constraints as a system of linear inequalities.

Step 3: Graph the system (the set of feasible points) and find the corner points.

Step 4: Evaluate the objective function at each corner point. The largest (or smallest) of these is the solution.

Example 3. The minimum value of $z = 15$ occurs at the point (5, 0)

Example 4. 20 racing skates and 16 figure skates should be produced to maximize profit at $492

CHAPTER 12 Sequences; Induction; the Binomial Theorem

Section 12.1: Sequences

p. 334. the set of positive integers

Example 1. $\left\{\frac{1}{3}, \frac{4}{5}, \frac{9}{7}, \frac{16}{9}, \frac{25}{11}, \frac{36}{13}\right\}$

Example 2. $\{-2, 4, -6, 8, -10, 12\}$

Example 3. $\{1, 4, 3, 16, 5, 36\}$

Example 4. **(a)** $\{a_n\} = \left\{\frac{1}{2^n}\right\}$ **(b)** $\{b_n\} = \{2n+3\}$ **(c)** $\{c_n\} = \left\{(-1)^n\right\}$ **(d)** $\{d_n\} = \{n^2+2\}$

p. 335.

$$0! = 1 \qquad 1! = 1$$
$$n! = n(n-1)\cdot \ldots \cdot 3 \cdot 2 \cdot 1 \qquad \text{if } n \geq 2$$

Example 5. 120; 720; $n(n-1)!$

Example 6. $\{5, 10, 20, 40, 80\}$

Example 7. $\{1, 1, 2, 3, 5\}$

Example 8. **(a)** $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \ldots + \frac{n}{n+1}$ **(b)** $-1+0+3+8+\ldots+(n^2-1)$

Example 9. (a) $\sum_{k=1}^{5}\left(\frac{1}{k}\right)^2$ **(b)** $\sum_{k=1}^{n}(2k-1)$

p. 337.

Properties of Sequences

If $\{a_n\}$ and $\{b_n\}$ are two sequences and c is a real number, then

$$\sum_{k=1}^{n}(ca_k) = ca_1 + ca_2 + \cdots + ca_n = c(a_1 + a_2 + \cdots + a_n) = c\sum_{k=1}^{n} a_k \quad \textbf{(1)}$$

$$\sum_{k=1}^{n}(a_k + b_k) = \sum_{k=1}^{n} a_k + \sum_{k=1}^{n} b_k \quad \textbf{(2)}$$

$$\sum_{k=1}^{n}(a_k - b_k) = \sum_{k=1}^{n} a_k - \sum_{k=1}^{n} b_k \quad \textbf{(3)}$$

$$\sum_{k=j+1}^{n} a_k = \sum_{k=1}^{n} a_k - \sum_{k=1}^{j} a_k \quad \text{where } 0 < j < n \quad \textbf{(4)}$$

Formulas for Sums of Sequences

$$\sum_{k=1}^{n} c = \underbrace{c + c + \cdots + c}_{n \text{ terms}} = cn \quad c \text{ is a real number} \quad \textbf{(5)}$$

$$\sum_{k=1}^{n} k = 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2} \quad \textbf{(6)}$$

$$\sum_{k=1}^{n} k^2 = 1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6} \quad \textbf{(7)}$$

$$\sum_{k=1}^{n} k^3 = 1^3 + 2^3 + 3^3 + \cdots + n^3 = \left[\frac{n(n+1)}{2}\right]^2 \quad \textbf{(8)}$$

Example 10. (a) 330 **(b)** 46 **(c)** 3960 **(d)** 17,952

p. 338. $A_n = \left(1+\frac{r}{N}\right)A_{n-1} + P$

Example 11. (a) $A_n = \left(1+\frac{.04}{12}\right)A_{n-1} + 60$ **(c)** No. On March 1st she has $567.05

p. 338. $A_n = \left(1+\frac{r}{12}\right)A_{n-1} - P$

Example 12. (a) 18,058.03 **(b)** 19 payments **(c)** 39 Months; $20,844.33

Section 12.2: Arithmetic Sequences

p. 340. $a_{n-1} + d$; first term; common difference

Example 1. (a) 4; -2 **(b)** 3; 4 **(c)** -1; -3

Exploration 1. $a_1+3d; a_1+4d; a_1+3d; a+(n-1)d$

p. 341. $a+(n-1)d$

Example 2. 66

Example 3. $6; a_n = a_{n-1}+4; a_n = 4n+2$

p. 341. $\frac{n}{2}\left[2a_1+(n-1)d\right]; \frac{n}{2}\left(a_1+a_n\right)$

Example 4. (a) $2n(2+n)$ **(b)** 1305

Example 5. 2360 seats

Section 12.3: Geometric Sequences; Geometric Series

Exploration 1. (1) (a) Arithmetic; $\{a_n\}=\{2n-1\}$ **(b)** Not arithmetic; $\{a_n\}=\{3^{n-1}\}$ **(c)** Arithmetic; $\{a_n\}=\{n\}$ **(d)** Not arithmetic; $\{a_n\}=\{2^{n-1}\}$ **(2)** They increase by a ratio

p. 343. ra_{n-1}; first term; common ratio

Example 1. (a) $2; 4$ **(b)** $9; 3$ **(c)** $2; 6$

Exploration 2. $a_1r^3; a_1r^4; a_1r^{n-1}$

p. 344. a_1r^{n-1}

Example 2. (a) $3\left(\frac{2}{3}\right)^8$ **(b)** $a_n=\left(\frac{2}{3}\right)a_{n-1}$

p. 345. $a_1+a_1r+a_1r^2+\ldots+a_1r^{n-1}; a_1r^{k-1}; a_1\cdot\frac{1-r^n}{1-r}, r\neq 0,1$

Example 3. (a) $S_n=-\frac{3}{2}\left(1-3^n\right); 9840$ **(b)** $\frac{7629394531}{30517578125}$

p. 345. a_1r^{k-1}

p. 346. $\frac{a_1}{1-r}$

Example 4. (a) $\frac{3}{2}$ **(b)** $0.\bar{8}=0.8+0.08+0.008+\ldots=\frac{0.8}{1-0.1}=\frac{8}{9}$

p. 346. $P\dfrac{(1+i)^n-1}{i}$

Example 5. No, she will not have enough money.

Example 6. \$286,033.27

Section 12.4: Mathematical Induction

p. 347. The statement is true for the natural number 1; The statement is true for some natural number k, and it can be shown to be true for the next natural number $k + 1$.

Example 1.

I: $n = 1$

$$\begin{aligned} 2(1)-1&=1^2\\ 1&=1 \end{aligned}$$

II: Assume true for some k.

$$1+3+5+\ldots+\left(2(k+1)-1\right)\stackrel{?}{=}(k+1)^2$$

$$1+3+5+\ldots+2k+2-1\stackrel{?}{=}(k+1)^2$$

$$1+3+5+\ldots2k+1\stackrel{?}{=}(k+1)^2$$

$$1+3+5+\ldots+2k-1+2k+1\stackrel{?}{=}k^2+2k+1$$

$$k^2+2k+1=k^2+2k+1$$

Conditions I and II are satisfied; by the Principle of Mathematical Induction, $1+3+5+\ldots+(2n-1)=n^2$ for all natural numbers n.

Example 2.

I: $n = 1$

$$\begin{aligned} 2(1)&=1(1+1)\\ 2&=2 \end{aligned}$$

II: Assume true for some k.

$$2\ +\ 4\ +\ \ldots+\ 2k+2(k+1)\ \stackrel{?}{=}(k+1)\left((k+1)+1\right)$$

$$k(k+1)+2(k+1)\ \stackrel{?}{=}(k+1)(k+2)$$

$$k^2+3k+2\stackrel{?}{=}(k+1)(k+2)$$

$$(k+1)(k+2)\stackrel{?}{=}(k+1)(k+2)$$

Conditions I and II are satisfied; by the Principle of Mathematical Induction, $2 + 4 + \ldots + 2n = n(n+1)$ for all natural numbers n.

Example 3.

I: $n = 1$

$2^1 > 1$

II: Assume true for some k.

$2^{k+1} > k+1$

$2^{k+1} = 2^1 \cdot 2^k > 2k = k + k \geq k+1$

Conditions I and II are satisfied; by the Principle of Mathematical Induction, $2^n > n$ for all natural numbers n.

Section 12.5: The Binomial Theorem

Exploration 1. (a) $x^2 + 4x + 4$ **(b)** $x^3 + 6x^2 + 12x + 8$ **(c)** $x^4 + 8x^3 + 24x^2 + 32x + 16$

p. 349. $\dfrac{n!}{j!(n-j)!}$

Example 1. (a) 4 **(b)** 15 **(c)** 10 **(d)** 10,272,278,170

p. 350.

$$\binom{n}{0} = 1 \qquad \binom{n}{1} = n \qquad \binom{n}{n-1} = n \qquad \binom{n}{n} = 1$$

Exploration 2. (1)

$$\begin{array}{c} 1 \\ 1 \quad 1 \\ 1 \quad 2 \quad 1 \\ 1 \quad 3 \quad 3 \quad 1 \\ 1 \quad 4 \quad 6 \quad 4 \quad 1 \\ 1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1 \\ 1 \quad 6 \quad 15 \quad 20 \quad 15 \quad 6 \quad 1 \\ 1 \quad 7 \quad 21 \quad 35 \quad 35 \quad 21 \quad 7 \quad 1 \\ 1 \quad 8 \quad 28 \quad 56 \quad 70 \quad 56 \quad 28 \quad 8 \quad 1 \end{array}$$

(2) Answers will vary. **(3)** See above triangle

p. 351.

$$(x+a)^n = \binom{n}{0}x^n + \binom{n}{1}ax^{n-1} + \cdots + \binom{n}{j}a^j x^{n-j} + \cdots + \binom{n}{n}a^n$$

$$= \sum_{j=0}^{n} \binom{n}{j} x^{n-j} a^j \qquad \textbf{(2)}$$

Example 2. (a) $x^5 + 10x^4 + 40x^3 + 80x^2 + 80x + 32$ **(b)** $81y^4 + 216y^3 + 216y^2 + 96y + 16$

p. 351. $\binom{n}{n-j} b^{n-j}(ax)^j$

Example 3. 103,680

Example 4. $20{,}412y^2$

CHAPTER 13 Counting and Probability

Section 13.1: Counting

p. 353. $A = B; A \subseteq B; A \subset B;\ A \subseteq B$ implies that A can equal B

Example 1. $\varnothing, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}$

p. 353. 2^n

Example 2. (a) 69 students **(b)** 31 students

p. 354. $n(A) + n(B) - n(A \cap B); n(A) + n(B); n(A_1) + n(A_2) + \ldots + n(A_n)$

Example 3. (a) 14,868 **(b)** 27,439

Example 4. 12

p. 345. $p \cdot q \cdot r \cdot \ldots$

Example 5. 15

Example 6. 17,576,000

Section 13.2: Permutations and Combinations

p. 357. ordered arrangements; n^r

Example 1. 17,576

Example 2. 15,600

p. 358. distinct; cannot be repeated; important; $\dfrac{n!}{(n-r)!}$

Example 3. (a) 210 **(b)** 6 **(c)** 311,875,200

Example 4. 120

Example 5. 48,228,180

p. 359. order; without; $r \le n; C(n,r)$

Example 6. 6

p. 359. distinct; cannot be repeated; not important; $\dfrac{n!}{(n-r)!r!}$

Example 7. (a) 3 **(b)** 20 **(c)** 1 **(d)** 1 **(e)** 2,598,960

Example 8. (a) 35 **(b)** 1800

p. 360. $\dfrac{n!}{n_1! \cdot n_2! \cdot \ldots n_k!}$

Example 9. (a) 15,120 **(b)** 280

Section 13.3: Probability

Exploration 1. Answers will vary.

Example 1. (a) $\{Heads, Tails\}$ **(b)** 0.5

p. 362. $\{e_1, e_2, \ldots, e_n\}; P(e_1)+P(e_2)+\ldots+P(e_n); 1$

Example 2. (a) **(b)** They add up to 1.

Outcome	Probability
1	$\frac{1}{6}$
2	$\frac{1}{6}$
3	$\frac{1}{6}$
4	$\frac{1}{6}$
5	$\frac{1}{6}$
6	$\frac{1}{6}$

Example 3.

Outcome	Probability
T	$\frac{1}{4}$
H	$\frac{3}{4}$

p. 363. $\dfrac{\text{Number of ways that E can occur}}{\text{Number of possible outcomes}}; \dfrac{m}{n}; \dfrac{n(E)}{n(S)}$

Example 4. $\dfrac{3}{8}$

Example 5. (a) Drawing an Ace of Hearts **(b)** Drawing a heart or an ace **(c)** $P(E)=\frac{1}{4}; P(F)=\frac{1}{13}$ **(d)** $\frac{1}{52}$ **(e)** $\frac{4}{13}$

p. 364. $P(E)+P(f)-P(E\cap F)$**;** Because it is double counted.

Example 6. 0.4

p. 364. $\varnothing$

Example 7. 0.5

p. 364. Not outcomes in the event *E*.

Exploration 2. (1) Not rolling a 6. $P(E)=\frac{1}{6}; P(\overline{E})=\frac{5}{6}; 1$

p. 365. $1-P(E)$

Example 8. 0.6

Example 9. $P(\overline{F})=\frac{12}{13}$

Example 10. 0.12

CHAPTER 14 A Preview of Calculus: The Limit, Derivative, and Integral of a Function

Section 14.1: Investigating Limits Using Tables and Graphs

Exploration 1. As *x* approaches 3, the function approaches 6
p. 366. $f(x)$; approaches; *N*; $x \neq c$; *f*; *N*; *c*; unequal
Exploration 1(2). $\lim_{x\to 3} f(x)=6$
Example 1. 11
Example 2a. 5
Example 2b. 3
Example 3. 0
Example 4. Does not exist
Example 5. $-\frac{3}{2}$

Section 14.2: Investigating Limits Using Tables and Graphs

Exploration 1(2). $\lim_{x\to c} 2=2$

Exploration 2(2). $\lim_{x\to c} x=c$

p. 370. *A*; *c*

Example 1. (a) 2 **(a)** 4 **(a)** -4 **(a)** $-\pi$

p. 370. $\lim_{x\to c} f(x)+\lim_{x\to c} g(x)$; $\lim_{x\to c} f(x)-\lim_{x\to c} g(x)$; $\left[\lim_{x\to c} f(x)\right]\left[\lim_{x\to c} g(x)\right]$

Example 2. (a) 5 **(b)** 1 **(c)** -3 **(d)** 7 **(e)** -3

p. 371. ac^n; $P(c)$

Example 5. (a) 250 **(b)** 18

p. 364. $\left[\lim_{x\to c} f(x)\right]^n$; $\sqrt[n]{\lim_{x\to c} f(x)}$

Example 4. (a) 4096 **(b)** $\sqrt{13}$ **(c)** 4

p. 372. $\dfrac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)}$

Example 5. (a) -1 **(b)** $\dfrac{4}{5}$ **(c)** 5

Example 6. 15

p. 373. $\lim_{x\to c} f(x)=f(c)$; the Limit of a Power; of a Root; the Limit of a Polynomial; Limit of a Quotient; factoring

Section 14.3: One-Sided Limits; Continuity

Exploration 1. (1a) 3 **(1b)** 2 **(1c)** 1 **(1d)** does not exist **(2a)** 1 **(2b)** -1 **(3a)** 1 **(3b)** 2

p. 374. left-hand limit; left; right-hand limit; right

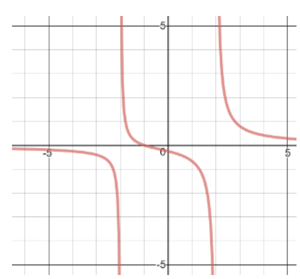
p. 375. equal; $\lim_{x\to c^-} f(x)$; $\lim_{x\to c^+} f(x)$

Example 1. (a) -2 **(b)** 2 **(c)** Does not exist

p. 375. domain; equals a number; $f(c)$; $f(c)$; $f(c)$; discontinuous

Example 2. (a) -1, 1 **(b)** continuous at all values except for -1 and 1

Example 3. (a) -2 and 2 **(b)** The limits at -2 and 2 do not exist **(c)**



Example 4. The function is continuous at all values except for $x = 4$.

p. 377.

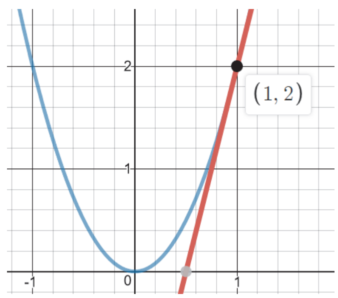
Library of Functions: Continuity Properties

Function	Domain	Property
Polynomial function	All real numbers	Continuous at every number in the domain
Rational function $R(x) = \frac{P(x)}{Q(x)}$, P, Q are polynomials	$\{x \mid Q(x) \neq 0\}$	Continuous at every number in the domain Hole or vertical asymptote where R is undefined
Exponential function	All real numbers	Continuous at every number in the domain
Logarithmic function	Positive real numbers	Continuous at every number in the domain
Sine and cosine functions	All real numbers	Continuous at every number in the domain
Tangent and secant functions	All real numbers, except odd integer multiples of $\frac{\pi}{2}$	Continuous at every number in the domain Vertical asymptotes at odd integer multiples of $\frac{\pi}{2}$
Cotangent and cosecant functions	All real numbers, except integer multiples of π	Continuous at every number in the domain Vertical asymptotes at integer multiples of π

Section 14.4: One-Sided Limits; Continuity

p. 378. $(c, f(c))$; $m_{\tan} = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$; $y - f(c) = m_{\tan}(x - c)$

Example 1. $y = 4x - 2$;



p. 378. $f'(c)$; $f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$

Example 2. 10

Example 3. 4

Example 4. $3c^2$

p. 379. $f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$

Example 5. 16π

Example 6. (a) 6 **(b)** 5 **(c)** 48 ft/sec **(d)** 16 ft/sec **(e)** 2.5 seconds **(f)** -80 ft/sec

Section 14.5: One-Sided Limits; Continuity

Exploration 1. (1-4) Answers will vary. **(5)** About 1/3

p. 383. $\frac{b-a}{n}$; u_i; $f(u_i)\Delta x$; $\sum_{i=1}^{n} f(u_i)\Delta x$; $\lim_{n \to \infty} \sum_{i=1}^{n} f(u_i)\Delta x$; $\int_a^b f(x)dx$

Example 1. (c) 26/3