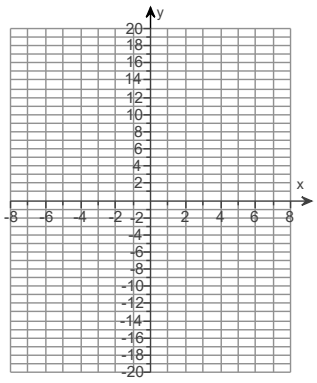
Student: _____ **Instructor:** Melanie Laber
Date: _____ **Course:** Precalculus (1) **Assignment:** PreCalc Semester 1 Test Out

1. Find all points having an x-coordinate of 3 whose distance from the point $(-1, -4)$ is 5.

The point(s) is(are) [ ].
(Type an ordered pair. Use a comma to separate answers as needed.)

2. (a) Find the intercepts of the graph of the equation.
(b) Test the equation for symmetry with respect to the x-axis, the y-axis, and the origin.
(c) Graph the equation by plotting points.

$y = x^3 - 4x$



(a) List the intercept(s). Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ○ **A.** The intercept(s) is/are [ ].
  (Type an ordered pair. Use a comma to separate answers as using radicals as needed.)
- ○ **B.** There are no intercepts.

(b) What are the results for the tests for symmetry? Select all that apply.

- ☐ **A.** The graph is symmetric with respect to the y-axis.
- ☐ **B.** The graph is symmetric with respect to the x-axis.
- ☐ **C.** The graph is symmetric with respect to the origin.
- ☐ **D.** The graph has no symmetry.

(c) Use the graphing tool to graph the function.

3. Solve the equation algebraically. Verify your solution using a graphic utility.

$$\frac{x+2}{2} + \frac{x+6}{3} = 7$$

Select the correct choice below and fill in any answer boxes in your choice.

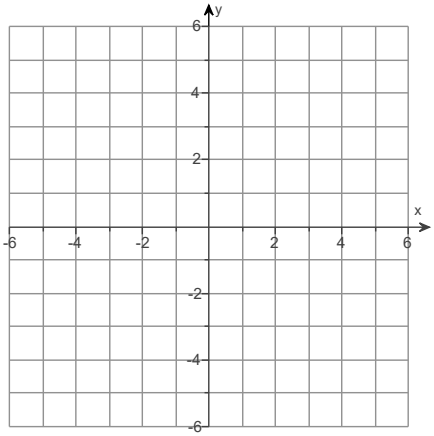
- ○ **A.** The solution set is $\{$ [ ] $\}$.
  (Simplify your answer. Use a comma to separate answers as needed.)
- ○ **B.** There is no real solution.

4. Write the standard form of the equation and the general form of the equation of the circle of radius $r = 1$ and center $(h,k) = (0,0)$. Graph the circle.

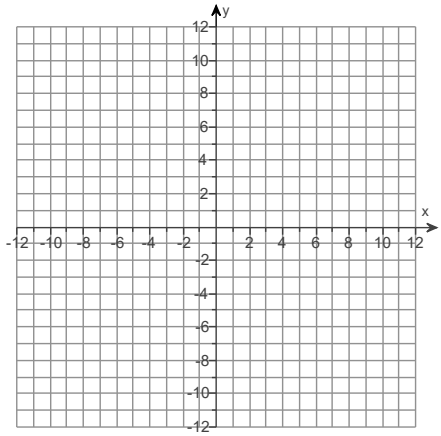
The standard form of the equation of this circle is [ ].

The general form of the equation of this circle is [ ].
(Simplify your answer.)

Use the graphing tool to graph a circle with center (0,0) and a radius of 1.



5. A circle has the equation $x^2 + y^2 = 49$.
(a) Find the center (h,k) and radius r of the circle.
(b) Graph the circle.
(c) Find the intercepts, if any, of the graph.



(a) The center of the circle is ☐.
(Type an ordered pair, using integers or decimals.)

The radius of the circle is ☐.
(Type an integer or a decimal.)

(b) Use the graphing tool to graph the circle.

(c) What are the intercepts? Select the correct choice below and, if necessary, fill in the answer box within your choice.

- ○ **A.** The intercept(s) is/are ☐.
  (Type an ordered pair. Use a comma to separate answers as radicals as needed.)
- ○ **B.** There are no intercepts.

6. Find the standard form of the equation of the circle with endpoints of a diameter at the points (1,8) and (−3,2).

Type the standard form of the equation of this circle.

☐ (Type an equation.)

7. Find the domain of the function.

$f(x) = \sqrt{2x - 14}$

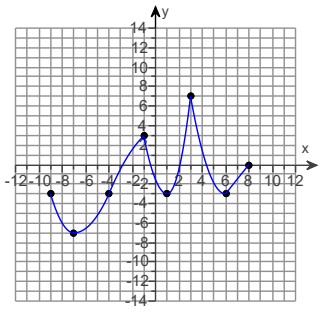
The domain is ☐. (Type your answer in interval notation.)

8. Find the difference quotient of f; that is, find $\frac{f(x+h) - f(x)}{h}$, $h \neq 0$, for the following function.

$f(x) = 3x + 3$

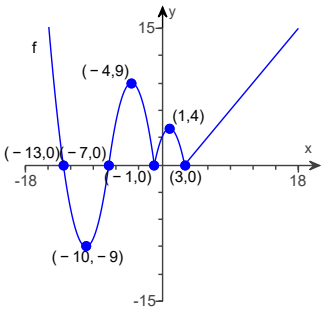
$\frac{f(x+h) - f(x)}{h} =$ ☐ (Simplify your answer.)

9. List the interval(s) on which f is decreasing.



The interval(s) on which f is decreasing is/are ☐.
(Type your answer in interval notation. Use a comma to separate answers as needed.)

10. Use the graph of the function f. Is there a local maximum at −4? If yes, what is it?



Select the correct choice below and, if necessary, fill in the answer box within your choice.

- ○ **A.** Yes, there is a local maximum at −4, and it is ☐.
  (Type an integer or a fraction.)
- ○ **B.** There is no local maximum at −4.

11. Let $h(x) = x^2 - 5x$.

(a) Find the average rate of change from 4 to 6.
(b) Find an equation of the secant line containing (4, h(4)) and (6, h(6)).

(a) The average rate of change from 4 to 6 is ☐. (Simplify your answer.)

(b) An equation of the secant line containing (4, h(4)) and (6, h(6)) is ☐.
(Type your answer in slope-intercept form.)

12. Which of the following functions has a graph that is symmetric about the y-axis?

Select all that apply.

☐ **A.** $y = \frac{1}{x}$

☐ **B.** $y = |x|$

☐ **C.** $y = \sqrt{x}$

☐ **D.** $y = x^3$

13. If $f(x) = \begin{cases} 5x - 2 & \text{if } -5 \le x \le 4 \\ x^3 - 4 & \text{if } 4 < x \le 6 \end{cases}$, find: (a) f(0), (b) f(1), (c) f(4), and (d) f(6).

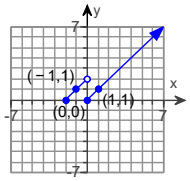
(a) $f(0) =$ ☐

(b) $f(1) =$ ☐

(c) $f(4) =$ ☐

(d) $f(6) =$ ☐

14. The graph of a piecewise-defined function is given. Write a definition for the function that best describes this graph.



$f(x) = \begin{cases} \square & \text{if } -2 \le x < \square \\ \square & \text{if } x \ge \square \end{cases}$

15. If $f(x) = \text{int}(2x)$, find:

(a) f(1.3) (b) f(2.9) (c) f(−1.7)

(a) $f(1.3) =$ ☐ (Simplify your answer.)

(b) $f(2.9) =$ ☐ (Simplify your answer.)

(c) $f(-1.7) =$ ☐ (Simplify your answer.)

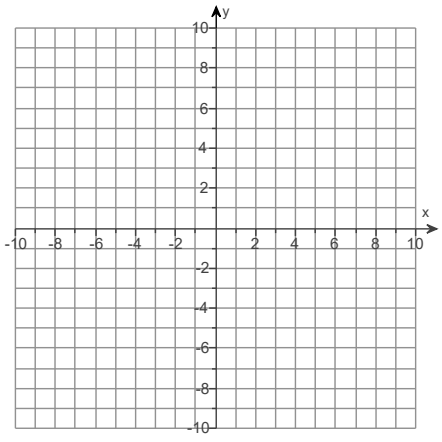
16. Starting with the graph of a basic function, graph the following function using the techniques of shifting, compressing, stretching, and/or reflecting. Find the domain and range of the function.

$g(x) = x^2 - 4$

Use the graphing tool to graph the equation.

The domain of g(x) is ☐.
(Type your answer in interval notation.)

The range of g(x) is ☐.
(Type your answer in interval notation.)



17. The point at which a company's profits equal zero is called the company's **break-even point**. Let R represent a company's revenue, let C represent the company's costs, and let x represent the number of units produced and sold each day.

$R(x) = 5x$
$C(x) = 3.5x + 750$

(a) Find the firm's break-even point; that is, find x so that $R = C$.
(b) Find the values of x such that $R(x) > C(x)$. This represents the number of units that the company must sell to earn a profit.

(a) $x =$ ☐ (Type a whole number.)

(b) Solve the inequality for x. Type the correct inequality symbol in the first answer box below, and type an integer in the second answer box.

x ☐ ☐

18. David has 400 yards of fencing and wishes to enclose a rectangular area.
(a) Express the area A of the rectangle as a function of the width W of the rectangle.
(b) For what value of W is the area largest?
(c) What is the maximum area?

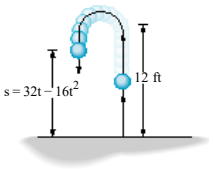
(a) $A(W) =$ ☐

(b) The area is largest for $W =$ ☐ (1) _____
(Simplify your answer.)

(c) The maximum area is ☐ (2) _____
(Simplify your answer.)

(1) ○ square yards. ○ yards. ○ cubic yards.
(2) ○ square yards. ○ cubic yards. ○ yards.

19. A ball is thrown vertically upward with an initial velocity of 32 feet per second. The distance s (in feet) of the ball from the ground after t seconds is $s = 32t - 16t^2$.
(a) At what time t will the ball strike the ground?
(b) For what time t is the ball more than 12 feet above the ground?



(a) The ball will strike the ground when t is ☐ seconds.

(b) The ball is more than 12 feet above the ground for the time t when ☐ < t < ☐.
(Simplify your answer.)

20. Determine whether the following function is a polynomial function. If the function is a polynomial function, state its degree. If it is not, tell why not. Write the polynomial in standard form. Then identify the leading term and the constant term.

$G(x) = 2(x-1)^2(x^2+4)$

Determine whether G(x) is a polynomial or not. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

○ **A.** It is not a polynomial because the variable x is raised to the ☐ power, which is not a nonnegative integer.
(Type an integer or a fraction.)

○ **B.** It is a polynomial of degree ☐.
(Type an integer or a fraction.)

○ **C.** It is not a polynomial because the function is the ratio of two distinct polynomials, and the polynomial in the denominator is of posit

Write the polynomial in standard form. Then identify the leading term and the constant term. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

○ **A.** The polynomial in standard form is $G(x) =$ ☐ with leading term ☐ and constant ☐.
(Simplify your answers. Use integers or fractions for any numbers in the expressions.)

○ **B.** The function is not a polynomial.

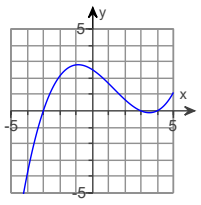
21. Form a polynomial whose zeros and degree are given.

Zeros: $-3, 3, 6$; degree: 3

Type a polynomial with integer coefficients and a leading coefficient of 1 in the box below.

$f(x) =$ ☐ (Simplify your answer.)

22. Determine whether the graph could be the graph of a polynomial function. If it could be, list the real zeros and state the least degree the polynomial can have.



Select the correct choice below and fill in any answer boxes within your choice.

- **A.** The graph shows a polynomial function. The real zero(s) is/are [ ]. The least degree the polynomial can have is [ ]
  (Use a comma to separate answers as needed. Round to the nearest integer as needed.)
- **B.** The graph does not show a polynomial function.

23. Analyze the polynomial function $f(x) = x^2(x - 10)$ using parts (a) through (h) below.

(a) Determine the end behavior of the graph of the function.

The graph of f behaves like $y =$ [ ] for large values of $|x|$.

(b) Find the x- and y-intercepts of the graph of the function.

The x-intercept(s) is/are [ ].
(Simplify your answer. Type an integer or a fraction. Use a comma to separate answers as needed. Type each answer only once.)

The y-intercept(s) is/are [ ].
(Simplify your answer. Type an integer or a fraction. Use a comma to separate answers as needed. Type each answer only once.)

(c) Determine the real zeros of the function and their multiplicity. Use this information to determine whether the graph crosses or touches the x-axis at each x-intercept.

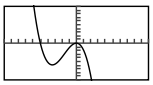
The real zero(s) of f is/are [ ].
(Simplify your answer. Type an integer or a fraction. Use a comma to separate answers as needed. Type each answer only once.)

The lesser zero of the function is of multiplicity [ ], so the graph of f (1) _____ the x-axis at x = [ ].

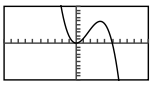
The greater zero of the function is of multiplicity [ ], so the graph of f (2) _____ the x-axis at x = [ ].

(d) Use a graphing utility to graph the function. The graphs are shown in the viewing window $X_{min} = -20$, $X_{max} = 20$, $X_{scl} = 2$, $Y_{min} = -250$, $Y_{max} = 250$, $Y_{scl} = 25$. Choose the correct graph below.

○ **A.**



○ **B.**



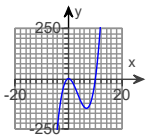
○ **C.**

(e) Approximate the turning points of the graph.

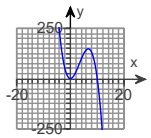
The turning point(s) of the graph is/are [ ].
(Type an ordered pair. Round each coordinate to the nearest hundredth as needed. Use a comma to separate answers as needed.)

(f) Use the information in parts (a) through (e) to draw a complete graph of the function by hand. Choose the correct graph below.

○ **A.**



○ **B.**



○ **C.**

(g) From the graph, find the range of the polynomial function.

The range of the polynomial function f is [ ].
(Type your answer in interval notation. Round to the nearest hundredth as needed.)

(h) Use the graph to determine where the function is increasing and where it is decreasing.

The function is increasing on the interval(s) [ ].
(Type your answer in interval notation. Round to the nearest hundredth as needed. Use a comma to separate answers as needed.)

The function is decreasing on the interval(s) [ ].
(Type your answer in interval notation. Round to the nearest hundredth as needed. Use a comma to separate answers as needed.)

(1) ○ touches ○ crosses (2) ○ touches ○ crosses

24. Use the given zero to find the remaining zeros of the function.

$f(x) = x^3 - 4x^2 + 25x - 100$; zero: $5i$

The remaining zero(s) of f is(are) [ ].
(Use a comma to separate answers as needed.)

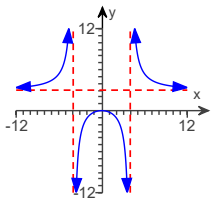
25. Find the domain of the following rational function.

$$R(x) = \frac{18x}{x + 20}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The domain of R(x) is {x | ☐}.
  (Type an inequality in the form x ≠ . Use integers or fractions for any numbers in the expression. Use a comma to separate answer
- **B.** The domain of R(x) is the set of all real numbers.

26. Use the graph shown to find the following.
(a) The domain and range of the function
(b) The intercepts, if any
(c) Horizontal asymptotes, if any
(d) Vertical asymptotes, if any
(e) Oblique asymptotes, if any



(a) What is the domain? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The domain of the function is {x | ☐}.
  (Type an inequality in the form x ≠ . Use integers or fractions for any numbers in the expression. Use a comma to separate answer
- **B.** The domain of the function is the set of all real numbers.

What is the range? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The range of the function is {y | ☐}.
  (Type an inequality. Use integers or fractions for any numbers in the expression. Use a comma to separate answers as needed.)
- **B.** The range of the function is the set of all real numbers.

(b) Find the intercepts, if there are any. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The intercept(s) is/are ☐.
  (Type an ordered pair. Use a comma to separate answers as needed.)
- **B.** There are no intercepts.

(c) Find the horizontal asymptotes, if there are any. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

- **A.** The function has one horizontal asymptote, ☐.
  (Type an equation.)
- **B.** The function has two horizontal asymptotes. The top asymptote is ☐ and the bottom asymptote is ☐.
  (Type equations.)
- **C.** The function has no horizontal asymptotes.

(d) Find the vertical asymptotes, if there are any. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

- **A.** The function has one vertical asymptote, ☐
  (Type an equation.)
- **B.** The function has two vertical asymptotes. The leftmost asymptote is ☐ and the rightmost asymptote is ☐
  (Type equations.)
- **C.** The function has no vertical asymptotes.

(e) Find the oblique asymptotes, if there are any. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

- **A.** The function has one oblique asymptote, ☐.
  (Type an equation. Type your answer in slope-intercept form.)
- **B.** The function has two oblique asymptotes. The oblique asymptote with negative slope is ☐ and the oblique asymptot
  (Type equations. Type your answers in slope-intercept form.)
- **C.** The function has no oblique asymptotes.

27. Find the vertical, horizontal, and oblique asymptotes, if any, for the following rational function.

$$T(x) = \frac{x^2}{x^4 - 1}$$

Find the vertical asymptotes. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

- **A.** The function has one vertical asymptote, ☐.
  (Type an equation. Use integers or fractions for any numbers in the equation.)
- **B.** The function has two vertical asymptotes. The leftmost asymptote is ☐ and the rightmost asymptote is ☐
  (Type equations. Use integers or fractions for any numbers in the equations.)
- **C.** The function has no vertical asymptote.

Find the horizontal asymptotes. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

- **A.** The function has one horizontal asymptote, ☐.
  (Type an equation. Use integers or fractions for any numbers in the equation.)
- **B.** The function has two horizontal asymptotes. The top asymptote is ☐ and the bottom asymptote is ☐.
  (Type equations. Use integers or fractions for any numbers in the equations.)
- **C.** The function has no horizontal asymptote.

Find the oblique asymptotes. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

- **A.** The function has one oblique asymptote, ☐.
  (Type an equation. Use integers or fractions for any numbers in the equation.)
- **B.** The function has two oblique asymptotes. The oblique asymptote with negative slope is ☐ and the oblique asymptot
  (Type equations. Use integers or fractions for any numbers in the equations.)
- **C.** The function has no oblique asymptote.

28. Evaluate each expression using the graphs of $y = f(x)$ and $y = g(x)$ shown below.

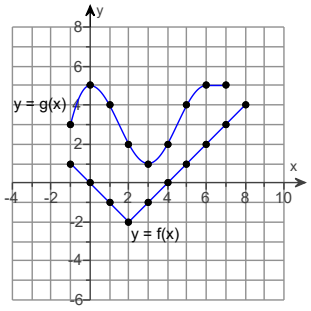
(a) $(g \circ f)(-1)$ (b) $(g \circ f)(0)$ (c) $(f \circ g)(-1)$ (d) $(f \circ g)(4)$

(a) $(g \circ f)(-1) =$ ☐
(Simplify your answer.)

(b) $(g \circ f)(0) =$ ☐
(Simplify your answer.)

(c) $(f \circ g)(-1) =$ ☐
(Simplify your answer.)

(d) $(f \circ g)(4) =$ ☐
(Simplify your answer.)



29. If $(3, -2)$ is a point on the graph of a one-to-one function f, which of the following points is on the graph of f^{-1}?

Choose the correct answer below.

- $(2, -3)$
- $(3,2)$
- $(-3,2)$
- $(-2,3)$

30. Consider the functions $f(x) = x^3 - 8$ and $g(x) = \sqrt[3]{x+8}$.
(a) Find f(g(x)).
(b) Find g(f(x)).
(c) Determine whether the functions f and g are inverses of each other.

(a) What is f(g(x))?

f(g(x)) = [ ] (Simplify your answer.)

Give any values of x that need to be excluded from f(g(x)). Select the correct choice below and fill in any answer boxes within your choice.

- **A.** $x \neq$ [ ]
  (Use a comma to separate answers as needed.)
- **B.** No values should be excluded from the domain.

(b) What is g(f(x))?

g(f(x)) = [ ] (Simplify your answer.)

Give any values of x that need to be excluded from g(f(x)). Select the correct choice below and fill in any answer boxes within your choice.

- **A.** $x \neq$ [ ]
  (Use a comma to separate answers as needed.)
- **B.** No values should be excluded from the domain.

(c) Are the functions f and g inverses of each other? Choose the correct answer below.

- Yes
- No

31. Consider the functions $f(x) = \dfrac{7x+4}{x+3}$ and $g(x) = \dfrac{3x-4}{7-x}$.
(a) Find f(g(x)).
(b) Find g(f(x)).
(c) Determine whether the functions f and g are inverses of each other.

(a) What is f(g(x))?

f(g(x)) = [ ] (Simplify your answer.)

Give any values of x that need to be excluded from f(g(x)). Select the correct choice below and fill in any answer boxes within your choice.

- **A.** $x \neq$ [ ]
  (Use a comma to separate answers as needed.)
- **B.** No values should be excluded from the domain.

(b) What is g(f(x))?

g(f(x)) = [ ] (Simplify your answer.)

Give any values of x that need to be excluded from g(f(x)). Select the correct choice below and fill in any answer boxes within your choice.

- **A.** $x \neq$ [ ]
  (Use a comma to separate answers as needed.)
- **B.** No values should be excluded from the domain.

(c) Are the functions f and g inverses of each other? Choose the correct answer below.

- No
- Yes

32. The function $f(x) = \frac{7x}{8x - 3}$ is one-to-one.

(a) Find its inverse and check your answer. (b) Find the domain and the range of f and f^{-1}.

(a) $f^{-1}(x) =$ [ ] (Simplify your answer.)

(b) Find the domain of f. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The domain is $\{x|x \geq$ [ ] $\}$.
  (Type integers or fractions. Use a comma to separate answers as needed.)
- **B.** The domain is $\{x|x \leq$ [ ] $\}$.
  (Type integers or fractions. Use a comma to separate answers as needed.)
- **C.** The domain is $\{x|x \neq$ [ ] $\}$.
  (Type integers or fractions. Use a comma to separate answers as needed.)
- **D.** The domain is the set of all real numbers.

Find the range of f. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The range is $\{y|y \leq$ [ ] $\}$.
  (Type integers or fractions. Use a comma to separate answers as needed.)
- **B.** The range is $\{y|y \geq$ [ ] $\}$.
  (Type integers or fractions. Use a comma to separate answers as needed.)
- **C.** The range is $\{y|y \neq$ [ ] $\}$.
  (Type integers or fractions. Use a comma to separate answers as needed.)
- **D.** The range is the set of all real numbers.

Now, find the domain of f^{-1}. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The domain is $\{x|x \geq$ [ ] $\}$.
  (Type integers or fractions. Use a comma to separate answers as needed.)
- **B.** The domain is $\{x|x \neq$ [ ] $\}$.
  (Type integers or fractions. Use a comma to separate answers as needed.)
- **C.** The domain is $\{x|x \leq$ [ ] $\}$.
  (Type integers or fractions. Use a comma to separate answers as needed.)
- **D.** The domain is the set of all real numbers.

Find the range of f^{-1}. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The range is $\{y|y \neq$ [ ] $\}$.
  (Type integers or fractions. Use a comma to separate answers as needed.)
- **B.** The range is $\{y|y \geq$ [ ] $\}$.
  (Type integers or fractions. Use a comma to separate answers as needed.)
- **C.** The range is $\{y|y \leq$ [ ] $\}$.
  (Type integers or fractions. Use a comma to separate answers as needed.)
- **D.** The range is the set of all real numbers.

33. Solve the equation.

$3^{-x} = 81$

The solution set is $\{$ [ ] $\}$.
(Type an integer or a simplified fraction. Use a comma to separate answers as needed.)

34. Change the logarithmic statement to an equivalent statement involving an exponent.

ln $11 = x$

The equivalent exponential statement is [ ]. (Type an equation.)

35. Solve the following equation. Write the answer in terms of the natural logarithm.

$e^{8x} = 3$

The solution set is $\{$ [ ] $\}$.
(Type an exact answer in simplified form. Use integers or fractions for any numbers in the expression. Use a comma to separate answers as needed.)

36. Solve the equation.

$$\log_5\left(x^2+2\right)=2$$

Change the given logarithmic equation to exponential form.

(Type an equation. Do not simplify.)

The solution set is $\{\ \}$.
(Type an exact answer in simplified form. Use a comma to separate answers as needed.)

37. Writing $\log_a u - \log_a v + 4\log_a w$ as a single logarithm results in which of the following expressions?

Choose the correct answer below.

- $\log_a\left(\frac{u}{vw^4}\right)$
- $\log_a\left(\frac{uw^4}{v}\right)$
- $\log_a(u-v+4w)$
- $\log_a\left(\frac{4uw}{v}\right)$

38. Solve the following logarithmic equation.

$$2\log_2(x-5)+\log_2 2=3$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The solution set is $\{\ \}$.
  (Simplify your answer. Type an exact answer. Use a comma to separate answers as needed.)
- **B.** There is no solution.

39. Solve the following exponential equation. Express irrational solutions in exact form.

$$6^{2x}+6^x-56=0$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The solution set is $\{\ \}$.
  (Simplify your answer. Use a comma to separate answers as needed. Use integers or fractions for any numbers in the expression. Type an exact answer, using radicals as needed.)
- **B.** There is no solution.